Strategic Analysis and Design of Multilayer Networks with a New Model of Interdependency

Arun Sen
Computer Science Program
School of Computing, Informatics and Decision Systems Engineering
Arizona State University

(On sabbatical at Wroclaw University of Technology, Poland/INRIA, Lille, France /Delft University, The Netherlands)
Background

The U.S. power delivery system is remarkably complex. It is a network of substations, transmission lines, distribution lines, and other components that people can see as they drive around the country; it also includes the less visible devices that sense and report on the state of the system, the automatic and human controls that operate the system, and the intricate web of computers and communication systems that tie everything together.
Example

Basic Structure of the Electric System

Color Key:
Blue: Transmission
Green: Distribution
Black: Generation

Generating Station
Generator Step Up Transformer
Transmission
Customer 138kV or 230kV
Transmission Lines
500, 345, 230, and 138 kV
Substation Step-Down Transformer
Primary Customer 13kV and 4 kV
Secondary Customer 120V and 240V
Subtransmission Customer 26kV and 69kV
Example (Continued)
Background
Since the Northeast Blackout of 1965, there has been an increasing integration of the power and telecommunications infrastructures. In particular, power systems have become increasingly dependent on the proper operation of supporting communication systems; failures in these supporting communications systems can result in system-wide blackouts. Blackouts that have been either directly caused by or aggravated by communication system failures have occurred in Europe as well as North America. Two clear examples of blackouts involving communication system elements have been experienced in the El Paso Electric (EPE) system and the Hydro-Québec system. In the EPE system, load was lost when a set of phase angle comparison relays improperly isolated a 345-kV transmission line. The improper operation of the relays was based on calculations using an incorrect communications latency value [2]. In the Hydro-Québec system, load was lost when a special protection system (SPS) experienced a single point failure in the supporting communications system [3]. In both cases, the loss of load could have been minimized if the interactions between the power and telecommunications infrastructures had been analyzed systematically. One of the reasons that this analysis was not performed is that there are limited tools for the systematic analysis of infrastructure interactions.
Critical infrastructures of nations, such as power and communication (but by no means only these two), are highly interdependent.

Understanding the nature of interdependency is extremely important from the national security standpoint as well as economic well being of any nation.

Unfortunately, our understanding of such interdependency is very limited at this time.

Serious efforts must be undertaken to have a better understanding of such interdependency. Effort such as NSF RIPS/CRISP Program is a step in the right direction.
NSF RIPS
(Resilient Interdependent Infrastructure Processes and Systems)

Background
One of the goals of the RIPS program is to increase resilience in Interdependent Critical Infrastructures

- How do you measure robustness/resiliency of ICIs?
  - Is there a metric to measure robustness/resiliency?
    - If there is no such metric, maybe a metric should be defined to measure robustness/resiliency
    - Without a metric it may be impossible to make a statement about how secure or vulnerable our integrated power-communication infrastructure is
    - With such a metric it maybe possible to make a statement that robustness/resiliency of our current ICI’s is at level X
    - If level X resiliency is inadequate, how to augment the ICI to reach level Y with least cost?
How do you measure robustness/resiliency of ICIs?

- Is there a metric to measure robustness/resiliency?
  - The problem is not that there is no metric to measure robustness/resiliency, the problem is that there are far too many of them (just like the standards!)
  - Everyone has their own favorite way of measuring “robustness” of a system. Some measure it in terms of the size of the “giant” component, some measure it in terms of network connectivity, some measure it in terms of MTBF (mean time between failure), MTTR (mean time to repair) and the list goes on and on
  - There has to be an universally acceptable agreement as to how “robustness” of ICIs should be measured.
If indeed there is no such known technique to measure resilience/vulnerability of ICIs, maybe efforts should be made to develop such techniques.

Similar examples

- There is a way to measure strength of a hurricane – Sandy was a category 3 hurricane.
- Hurricane has multiple attribute such as wind speed, storm surge etc., just like ICIs.
- Military readiness level is also categorized.
- The notion of “reliability” of a system has some similarity to the notion of resilience/vulnerability.
  - Reliability Theory is a very well established discipline.
- To the best of our knowledge there is no such theory of robustness/vulnerability or resilience.
- Just as it is possible to measure the strength of a hurricane or preparedness of a military, there should be a way to measure vulnerability and/or resilience of ICIs.
In the previous slides we made some observations and raised some questions regarding vulnerability/resilience of ICIs.

What type of analysis will be necessary to answer those questions?

Are those questions worthy enough to spend time and effort to find answers?
We believe that analysis both at the “microscopic” and “macroscopic” analysis is essential.

What is our notion of microscopic and macroscopic analysis?

What type of questions can be answered through microscopic analysis?

What type of questions can be answered through macroscopic analysis?

Is one type of analysis adequate to answer the questions raised earlier?
The goal of the RIPS program is to “create theoretical frameworks and multidisciplinary computational models of interdependent infrastructure systems”

What are the current models of interdependent infrastructure systems?
- With particular reference to interdependence between power and communication networks

In the past few years quite a few models of interdependent infrastructure systems have been proposed.
Unfortunately, most of the models do not capture the physical reality of interdependent power-communication infrastructure.

To the best of our knowledge, no effort has been undertaken to validate any one of them.

Although interdependency between the entities of a single layer network is better understood than the interdependency between the entities of a multilayer network, it is still not complete.

Case in point: Although infrequent, cascading failure events in Power Grid take place on a fairly regular basis.
Interdependent Power-Communication Infrastructure

- The topic is too important to be ignored
- We need to have a much better understanding of the topic
- We need to have the right model that captures the physical reality fairly accurately.
- A model can be considered “the right model” only if can be validated

- Difficulty in Validation
  - Difficulty in obtaining data
  - Cascading failures are infrequent

- Data Collection - although difficult, may not be impossible
  - Research support organizations, such as the NSF can help
Interdependent critical infrastructure systems comprising of Power and Communication Networks

More than 90 percent of the U.S. power grid is privately owned and regulated by the states, making it challenging for the federal government to address potential vulnerabilities to its operation, and perhaps especially its vulnerability to terrorist attack.
Modeling Interdependent Infrastructure Networks

Limitations of Existing Models & Proposed New Model
Multilayered Complex Network
Multiplicity of Models

• Many models have been proposed in the last few years
• For example:
  ▫ Rosato Model (2008)
  ▫ Buldyrev Model (2010)
  ▫ Peeta Model (2011)
  ▫ Castet Model (2012)
  ▫ Liu Model (2012)
  ▫ Modiano Model (2013)
Multiplicity of Models

The Rosato Model (2008)

Figure 1  The graph corresponding to the Italian high-voltage (380 kV) transmission grid resulting from the available data.

Notes: Source S nodes are square, Load L nodes triangles and Junction J nodes are black circles.

Figure 4  The high-bandwidth backbone of the internet network dedicated to linking Italian universities and research institutions (GARR)
Multiplicity of Models

The Rosato Model (2008)

- Realistic modeling of Power Network (PN) and Communication Network (CN)

- Effect of perturbation of PN on CN is analyzed based on a coupling parameter

- The impact of CN on PN is not analyzed

- The coupling parameter is not validated and is assumed
A component in a composite graph is connected if any two nodes have at least one blue path and one green path connecting them.
Multiplicity of Models

The Buldyrev Model (2010)

- Fault propagation with both intra link connection and inter link interdependencies in consideration

- Network robustness --- maximum number of node removal from one network to get at least one giant connected cluster (percolation threshold)

- Nodes in PN not designated as generator, substations or load and in CN not designated as routers or control centers

- Actual working of SCADA system in CN needs to be considered in modeling interdependency
Multiplicity of Models

The Buldyrev Model (2010)

The transients and readjustments of the system can be local in effect or can involve components far away, so that a component disconnection or failure can effectively increase the loading of many other components throughout the network. In particular, the propagation of failures is not limited to adjacent network components.

- In Probability in the Engineering and Informational Sciences, 2005
- A LOADING-DEPENDENT MODEL OF PROBABILISTIC CASCADING FAILURE
- Ian Dobson
- Electrical & Computer Engineering Department
- University of Wisconsin-Madison
Multiplicity of Models

The Liu Model (2012)
Multiplicity of Models

The Liu Model (2012)

- Effect of cyber intrusions in SCADA and EMS system on PN is analyzed through realistic test beds
- The experiments are confined to small domain
- Large cascades of failure owing to this effect is not analyzed
Multiplicity of Models

WSCC 9 Bus System

Node weighted graph model with generator and load weights

Edge weighted graph model with power flow on the transmission links
Limitations of Current Models

- Dependencies that exist between the entities of the Power and Communication networks are often complex, involving a combination of conjunctive and disjunctive terms representing the entities of these two types of networks.

- Most of the proposed interdependency models are unequipped to capture such complex interdependencies.
Example

Basic Structure of the Electric System

**Color Key:**
- Blue: Transmission
- Green: Distribution
- Black: Generation

- **Generating Station**
- **Generator Step Up Transformer**
- **Transmission Lines**
  - 500, 345, 230, and 138 kV
- **Transmission Customer**
  - 138kV or 230kV
- **Substation Step-Down Transformer**
- **Subtransmission Customer**
  - 26kV and 69kV
- **Primary Customer**
  - 13kV and 4 kV
- **Secondary Customer**
  - 120V and 240V
Example (Continued)

$$b_1 \leftarrow a_1 a_2 a_3 a_5 a_6 a_7 a_8 a_9 a_{10} a_{11} + a_{12} a_{13}$$
An example of limitation of the current models

- Power Network Entity $\text{PNE}_a$ (say, a generator, substation, transmission line, load) is “alive” if communication network entities
  - $\text{CNE}_b$ and $\text{CNE}_c$ and $\text{CNE}_d$ are alive, OR
  - $\text{CNE}_e$ and $\text{CNE}_f$ are alive, OR
  - $\text{CNE}_g$ is alive

- Examples of communication network entities may include routers, cell towers, fiber optic lines, optical signal amplifiers.
An example of limitation of the current models (continued)

- We introduce a new model to capture such complex dependencies using Boolean logic.
- We express the dependency relation in the example in the previous slide in the following way:

\[
PNE_a \leftarrow CNE_b CNE_c CNE_d + CNE_e CNE_f + CNE_g
\]

- This dependency relation is a necessary but not sufficient condition for \( PNE_a \) to be “alive”.
Cascading Failures in Multi-layered Networks

- Failures in a multi-layered network can cascade from layer to layer.

- CNE’s such as routers can not operate without power and PNE’s such as Supervisory Control and Data Acquisition Systems (SCADA) can not operate without control signals received through communication network.
Cascading Failures in Multi-layered Networks

- Failures propagate in time steps.
- We denote PNE’s as type A entities and CNE’s as type B entities.
- Let Boolean variables ‘a’ and ‘b’ to indicate the states of the entities.
- Cascading failures reach a steady state after $K$ time steps.
Multilayer Complex Network System

Multi-layer Interdependent Networks as Closed Loop Feedback Control System

Steady State in a Multilayered Complex Network System corresponds to a “fixed point” in the system:

\[ f\left(A^K_d \cup B^K_d\right) = A^K_d \cup B^K_d \]
An Example

Power Network

\[ a_1 \leftarrow b_1 + b_2 \]
\[ a_2 \leftarrow b_1 b_3 + b_2 \]
\[ a_3 \leftarrow b_3 b_1 b_2 \]
\[ a_4 \leftarrow b_1 + b_2 + b_3 \]

<table>
<thead>
<tr>
<th>Entities</th>
<th>Time Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t_1</td>
</tr>
<tr>
<td>a_1</td>
<td>1</td>
</tr>
<tr>
<td>a_2</td>
<td>0</td>
</tr>
<tr>
<td>a_3</td>
<td>0</td>
</tr>
<tr>
<td>a_4</td>
<td>0</td>
</tr>
</tbody>
</table>

Communication Network

\[ b_1 \leftarrow a_1 + a_2 a_3 \]
\[ b_2 \leftarrow a_1 + a_3 \]
\[ b_3 \leftarrow a_1 a_2 \]

<table>
<thead>
<tr>
<th></th>
<th>t_1</th>
<th>t_2</th>
<th>t_3</th>
<th>t_4</th>
<th>t_5</th>
<th>t_6</th>
<th>t_7</th>
</tr>
</thead>
<tbody>
<tr>
<td>b_1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>b_2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>b_3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
This report focuses on measures that could:
1. Make the power delivery system less vulnerable to attacks,
2. Restore power faster after an attack,
3. Make critical services less vulnerable while the delivery of conventional electric power has been disrupted.
Implicative Interdependency Model (IIM)


- Identification of K Most Vulnerable Nodes in the Interdependent Networks
  (Published in IEEE NetSciCom 2014, an Infocom Workshop)

- Root Cause of Failure Analysis
  (Published in IEEE Milcom 2014)

- Progressive Recovery Problem
  (Published in CRITIS 2014)

- Entity Hardening Problem in Networks
  (Published in IEEE WIDN 2015, an Infocom Workshop)

- Robustness Analysis Problem
  (To be presented at CRITIS in October 2015)

- Smallest Pseudo Target Set Identification Problem
  (To be presented at IEEE Milcom in November 2015)

- Robustness Analysis with Incomplete or Incorrect Information
  (Currently under study)
Prob. 1: Vulnerable Node Identification

- **Problem:** Identification of $K$ most vulnerable entities in a multi-layered network.

- **Definition:** A set of entities in a multi-layered network is said to be the "most vulnerable" if failure of the $K$ entities induces failure of the largest number of other entities in the multi-layered network.

An Example

Power Network

\[ a_1 \leftarrow b_1 + b_2 \]
\[ a_2 \leftarrow b_1 b_3 + b_2 \]
\[ a_3 \leftarrow b_3 b_1 b_2 \]
\[ a_4 \leftarrow b_1 + b_2 + b_3 \]

Communication Network

\[ b_1 \leftarrow a_1 + a_2 a_3 \]
\[ b_2 \leftarrow a_1 + a_3 \]
\[ b_3 \leftarrow a_1 a_2 \]
Prob. 2: Root Cause of Failure

• Anatomy of Failures in Interdependent Networks:
  ▫ Introduction of an *event-induced failure* in the system
    • Natural disasters (Hurricanes, Earthquakes), or terrorist attacks
  ▫ Further *triggered failures* caused by *event-induced failures* due to the nature of interdependencies shared
  ▫ Further *triggered-failures* caused by *event-induced and triggered failures* due to the nature of interdependencies shared
  ▫ End of Cascade, no further failures in the system

• Objective of this study (Root Cause of Failure Analysis):
  ▫ From the *final failure set* (*event-induced + triggered failures*) identify the original *event-induced failure*

## An Example

### Power Network

\[
\begin{align*}
a_1 & \leftarrow b_1 + b_2 \\
a_2 & \leftarrow b_1 b_3 + b_2 \\
a_3 & \leftarrow b_3 b_1 b_2 \\
a_4 & \leftarrow b_1 + b_2 + b_3
\end{align*}
\]

### Communication Network

\[
\begin{align*}
b_1 & \leftarrow a_1 + a_2 a_3 \\
b_2 & \leftarrow a_1 + a_3 \\
b_3 & \leftarrow a_1 a_2
\end{align*}
\]

<table>
<thead>
<tr>
<th>Entities ( a_i )</th>
<th>( t_1 )</th>
<th>( t_2 )</th>
<th>( t_3 )</th>
<th>( t_4 )</th>
<th>( t_5 )</th>
<th>( t_6 )</th>
<th>( t_7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( a_4 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( b_3 )</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Vulnerable Node Identification vs. Root Cause of Failure

Vulnerable Node Identification Problem

Root Cause of Failure Problem
Prob. 3: Progressive Recovery Problem

• In an Interdependent Multi-layer Network System, failure of a set of $A$ and $B$ type entities initially (i.e. at time $t = 0$) can eventually lead to the failure of a much larger set of $A$ and $B$ type entities through the cascading failure process.

• In order to take the system back to its original state all the entities that failed at time $t = 0$ must be repaired.

• The entities can be fixed one after another in a sequential fashion.
Prob. 3: Progressive Recovery Problem

• Just as failure of one entity can lead to the failure of another entity, fixing of one entity can lead to the fixing of another entity

• Fixing of one entity brings some “utility” value to the system
  ▫ Failure of a number of power lines can cause a blackout to a large number of households. Fixing one power line can bring back power to some households.

• The sequence of fixing the originally failed entities will determine the system utility during the duration of the repair operation
Prob. 3: Progressive Recovery Problem

- Consider the following example of a set of IDR:

<table>
<thead>
<tr>
<th>Power Network</th>
<th>Communication Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1 \leftarrow \emptyset$</td>
<td>$b_1 \leftarrow a_1 a_2$</td>
</tr>
<tr>
<td>$a_2 \leftarrow \emptyset$</td>
<td>$b_2 \leftarrow a_1 + a_2$</td>
</tr>
<tr>
<td>...</td>
<td>$b_3 \leftarrow a_1$</td>
</tr>
</tbody>
</table>

- Failure of $(a_1, a_2)$ leads to the failure of $(b_1, b_2, b_3)$

- In order to return the system to its normal operational state both $a_1$ and $a_2$ must be repaired

- Repair sequence could be $(a_2, a_1)$ or $(a_1, a_2)$.
  - Which repair sequence should be used?
    - Does it matter?
Prob. 3: Progressive Recovery Problem

• Whether \( a_1 \) is repaired first and then \( a_2 \), or the other way around, will have an impact on “system utility”

• Utility of an entity \( a_i \), \( u(a_i) \) is defined as the “benefit” obtained when the entity \( a_i \) is made operational

• \( x_{a_i}(t) \): Indicator variable for entity \( a_i \) such that:
  \[
  x_{a_i}(t) = \begin{cases} 
  1 & \text{entity } a_i \text{ is operational at time } t \\
  0 & \text{otherwise}
  \end{cases}
  \]
Prob. 3: Progressive Recovery Problem

- **$SUIT(t)$**: System Utility at Instance of Time $t$
  
  \[ SUIT(t) = \sum_{a_i \in V(A)} u(a_i)x_{a_i}(t) + \sum_{b_j \in V(B)} u(b_j)x_{b_j}(t) \]

- **$SUOT(t)$**: System Utility Over Time interval 0 to $T$
  
  \[ SUOT[T] = \sum_{t=0}^{T} SUIT(t) \]

- Example:
  - $u(a_1) = 10, u(a_2) = 10$
  - $u(b_1) = 20, u(b_2) = 30, u(b_3) = 40$
Prob. 3: Progressive Recovery Problem

Example:

<table>
<thead>
<tr>
<th>Power Network</th>
<th>Communication Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1 \leftarrow \emptyset$</td>
<td>$b_1 \leftarrow a_1a_2$</td>
</tr>
<tr>
<td>$a_2 \leftarrow \emptyset$</td>
<td>$b_2 \leftarrow a_1 + a_2$</td>
</tr>
<tr>
<td>...</td>
<td>$b_3 \leftarrow a_1$</td>
</tr>
</tbody>
</table>

- If the repair sequence is $(a_2, a_1)$, then the system utility over time changes as follows.
- Fixing of $a_2$ leads to fixing of $b_2$. Since $u(a_2) = 10$, and $u(b_2) = 30$
  
  $SUIT(1) = 10 + 30 = 40$

- Now fixing $a_1$ leads to fixing of $b_1, b_3$. Since $u(a_1) = 10$, $u(b_1) = 20$,
  $u(b_3) = 40$

  $SUIT(2) = 10 + 30 + 10 + 20 + 40 = 110$

<table>
<thead>
<tr>
<th>Time step $(t)$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SUIT(t)$</td>
<td>0</td>
<td>40</td>
<td>110</td>
</tr>
<tr>
<td>$SUOT[T]$</td>
<td>0</td>
<td>40</td>
<td>150</td>
</tr>
</tbody>
</table>
Prob. 3: Progressive Recovery Problem

- If the repair sequence is \((a_1, a_2)\), then the system utility over time is as follows:

<table>
<thead>
<tr>
<th>Time step ((t))</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(SUIT(t))</td>
<td>0</td>
<td>80</td>
<td>110</td>
</tr>
<tr>
<td>(SUOT[T])</td>
<td>0</td>
<td>80</td>
<td>190</td>
</tr>
</tbody>
</table>

- In this example the second sequence is preferable over the first.
- Lesson learnt: Repair sequence matters!
- The goal of the progressive recovery problem is to identify the repair sequence such that the system utility over time \(SUOT[T]\) is maximized.
Prob. 3: Progressive Recovery Problem

- Problem statement:
  - Find the sequence in which the originally failed entities (i.e. the entities that failed at $t = 0$) should be repaired so that the total system utility is maximized.

Prob. 4: Entity Hardening Problem

- **Problem Domain:** Adversarial Setting (Attacker-Defender Scenario)
- **Adversary Knowledge:** All the Dependency Relations that govern the system
- **Adversary Intention:** Cause maximum damage to the system (maximize inoperable entities)
- **Adversary Resources:** Adversary can render inoperable at most $K$ entities of the system
- **Adversary Action:** Identify $K$ most vulnerable nodes in the system
Prob. 4: Entity Hardening Problem

- If defender takes no action then the adversary will destroy $K$ most vulnerable entities in the system that will cause maximum damage

- **Entity Defense:** Defender takes some action so that the attacker cannot destroy the entity

- If the defender has the resources to defend $K$ entities then the attacker cannot inflict any damage to the system

- If the defender does not have resources to defend $K$ entities, but say $K'$ entities, where $K' \leq K$, the defender has to decide which $K'$ entities should be defended so that the impact of attack is minimized
Prob. 4: Entity Hardening Problem

• The $K'$ entities that the defender decides to defend can no longer be rendered inoperable and will be considered as “Hardened” entities.

• Entity Hardening problem is to identify the $K'$ entities that should be defended by the defender so that impact of attack is minimized.

• The implication of hardening an entity is a change in the set of dependency relations.

• The dependency relations of the hardened entities can be removed from the set of dependency relations as these entities can no longer fail.
Prob. 4: Entity Hardening Problem

- **Assumption:** The attacker is unaware of the action taken by the defender, i.e. how many entities, or which entities have been hardened.

- As a consequence the attacker operates with the pseudo (original) set of dependency relations which may not be the real set of dependency relations that describes the system (after the hardening process).

- The goal of the entity hardening problem is to identify the set of $K'$ entities whose hardening would minimize the impact of attack.
Solution Approach

• Complexity Analysis of individual cases of dependency relations
  ▫ Most general form of the dependency relation:
    \[ a_i \leftarrow b_j b_k b_l + b_m b_n + b_p \]
  ▫ In the general form:
    • No. of Min-terms are arbitrary
    • Size of Min-terms are arbitrary
  ▫ We consider four special cases for each problem:

<table>
<thead>
<tr>
<th>Case</th>
<th>No. of Min-terms</th>
<th>Size of Min-terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Case 2</td>
<td>Arbitrary</td>
<td>1</td>
</tr>
<tr>
<td>Case 3</td>
<td>1</td>
<td>Arbitrary</td>
</tr>
<tr>
<td>Case 4 (General)</td>
<td>Arbitrary</td>
<td>Arbitrary</td>
</tr>
</tbody>
</table>
Solution Approach

• Computation of optimal solution for each type of dependency relations
  ▫ Using Integer Linear Programming (if NP-Complete)

• Development of a Approximate/Heuristic algorithm to compute solution for dependency relations proven to be NP-Complete

• Comparison of Optimal vs. Approximate / Heuristic approach with experimental results using both real and synthetic data

Prob. 5: Smallest Pseudo-Target Set Identification Problem (STASIP) for Targeted in Interdependent Power-Communication Networks

In a multi-layered network with entities $A \cup B$, we define a set of entities $A'' \cup B''$ as pseudo target set for a targeted attack against a real target set $A' \cup B'$, if $A_d^0 = A''$ and $B_d^0 = B''$ implies $A_d^p \supset A'$ and $B_d^p \supset B'$, where $A'$, $A'' \subseteq A$ and $B'$, $B'' \subseteq B$. 
Prob. 5: Smallest Pseudo-Target Set Identification Problem (SPTSIP) for Targeted in Interdependent Power-Communication Networks

In a multi-layered network with entities $A \cup B$, we define a set of entities $A'' \cup B''$ as pseudo target set for a targeted attack against a real target set $A' \cup B'$, if $A^0_d = A''$ and $B^0_d = B''$ implies $A^p_d \supseteq A'$ and $B^p_d \supseteq B'$, where $A', A'' \subseteq A$ and $B', B'' \subseteq B$.

The goal of the SPTSIP is to identify the pseudo target set of the smallest size. In other words, identify the subsets $A^0_d \subseteq A$, $B^0_d \subseteq B$, such that $|A^0_d \cup B^0_d|$ is smallest and $A^p_d \supseteq A'$ and $B^p_d \supseteq B'$. 
Prob. 6: Least Cost Robustness Enhancement

- Definition: Robustness Level of an interdependent network is measured in terms of the fewest number of entities whose failure will trigger failure of all (or a certain percentage) of the entities in the interdependent network.

- The goal of the Least Cost Robustness Enhancement problem is to identify the way to take the network from Robustness Level X to Robustness Level Y, with least cost.
Experimental Data Sets

• Data Collection
  ▫ CNE Data - Data of Cell Towers, Fiber Lit Buildings and Fiber Routes was collected from Geo-tel (http://www.geo-tel.com/) for Maricopa County.
Experimental Data Sets

- **Data Collection**
  - PNE data – Data of Power Plants and Transmission Lines was collected from Platts ([http://www.platts.com/](http://www.platts.com/)) for Maricopa County.
Experimental Data Sets

- Data from Maricopa County
- Power Network (PNE):
  - Power plants: 70, Transmission Lines: 470
- Communication Network (CNE):
  - Cell Towers: 2960, Fiber-lit building: 7100,
  - Fiber Links: 42,723
Future Directions

• Discovery of Dependency Relations
• Deterministic Dependency Relations vs. Probabilistic Dependency Relations
• Exploration of scale and granularity of entities for interdependent multilayer network analysis
• Generalization from Binary (operational/non-operational) state of entities to $n$-ary states
• Identification of robustness and resiliency metrics for interdependent networks
• Phasor placement problem taking into account interdependency between the networks
Future Directions

• “Connected Component” Analysis:
  - Generalization of the concept of Connected Component of a Graph, $G = (V, E)$ (single layer), to Multi-layer Interdependent Network $(G_1, G_2, ..., G_m, R)$

• “Islanding” in Multi-Layer Networks
  - Generalization of the concept of an “island” in a power network, $G = (V, E)$ (single layer), to a Multi-layer Interdependent Network $(G_1, G_2, ..., G_m, R)$
Thank You!