Modeling time-varying multilayer networks (and beyond?)

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LIP6, Paris, France, October 09, 2014

LNCC: National Laboratory for Scientific Computing

- **Research unit** - Brazilian Ministry of Science, Technology and Innovation
- ~**60 dedicated researchers** (public servants) from different domains: engineering, computer science, applied math, physics, and biology
- **Interdisciplinary research**
  - Graduate program in **computational modeling** (~100 PhD/MSc students)
  - ~75 postdocs and research assistants
  - a lot of external **cooperative R&D** work (in Brazil and abroad)

Where are we???

Petrópolis, RJ is in the mountains about 70 km (~45 miles) away from **Rio**

LNCC:
Computational modeling

Laboratório Nacional de Computação Científica

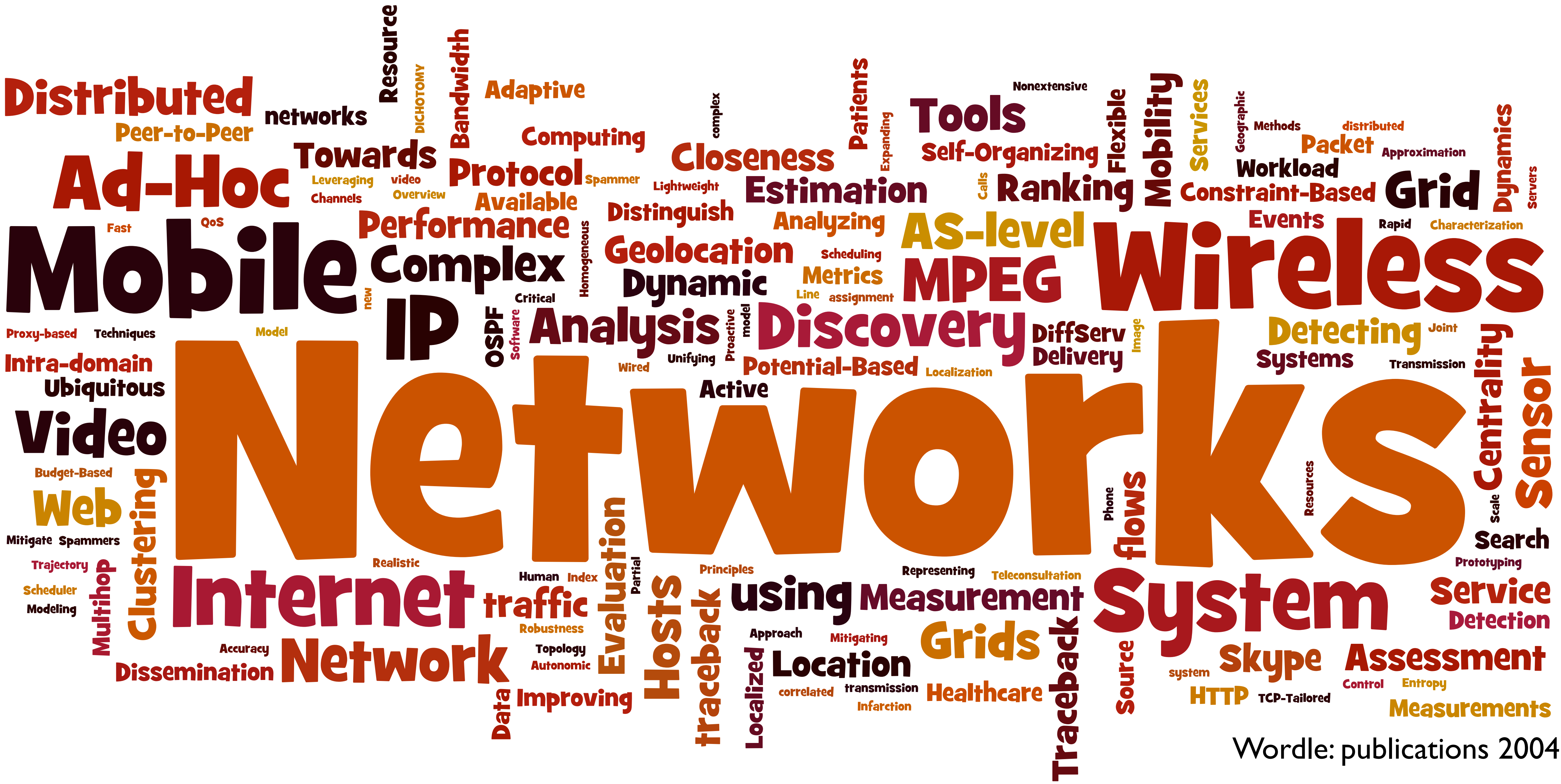
- **Basic research**
  - Numerical methods
  - First principals based modeling
  - Data-driven modeling

- **Applied research**
  - Medicine assisted by computing
  - Computational biology
  - Oil and gas

...

- Distributed and high-performance computing
- Complex networking, softwares, and databases

Wordle cloud of what I do...



Wordle: publications 2004-2014

Co-authorship

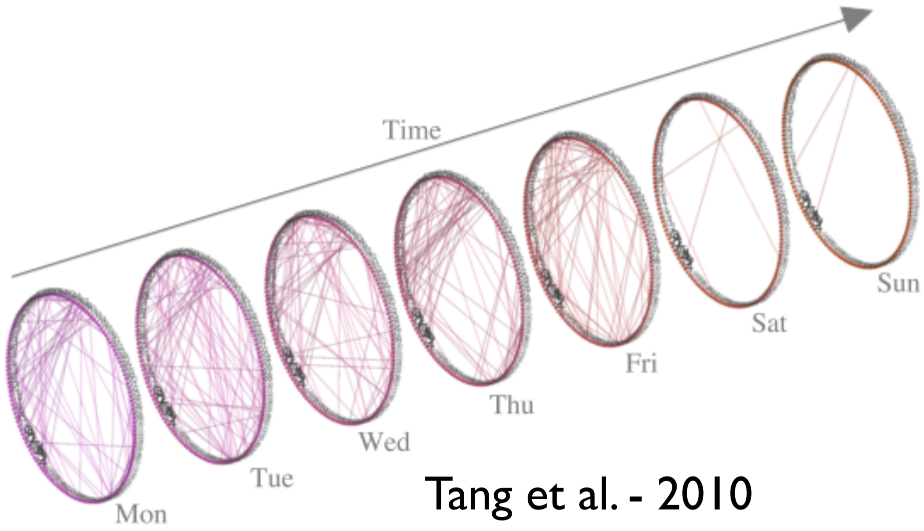
Academy in Brazil: IME, LNCC, PUC-Minas, PUC-PR, PUC-Rio, UFG, UFMG, UFJF, UFRJ, UNICAMP, UTFPR;

Academy abroad: University of Hamburg, Fraunhofer FOKUS (Germany); UBA (Argentina); Boston University (USA); LIP6/CNRS, ENS-Lyon, INRIA (France); Technion (Israel); NCKU (Taiwan);

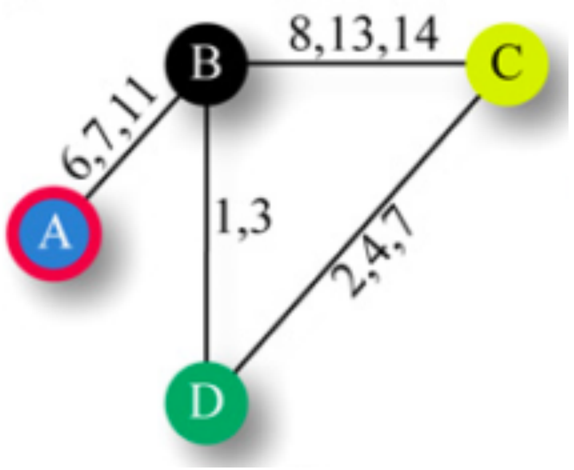
Industry: Google Engineering, Yahoo! Research, Deutsche Telekom Labs, Oi Telecom.

Dynamic Networks

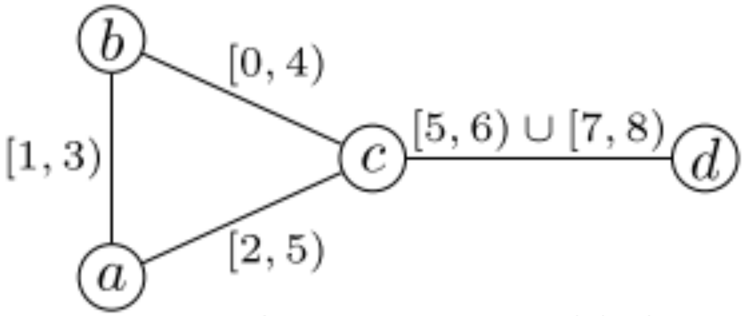
Represented by **Time-Varying Graphs (TVGs)**



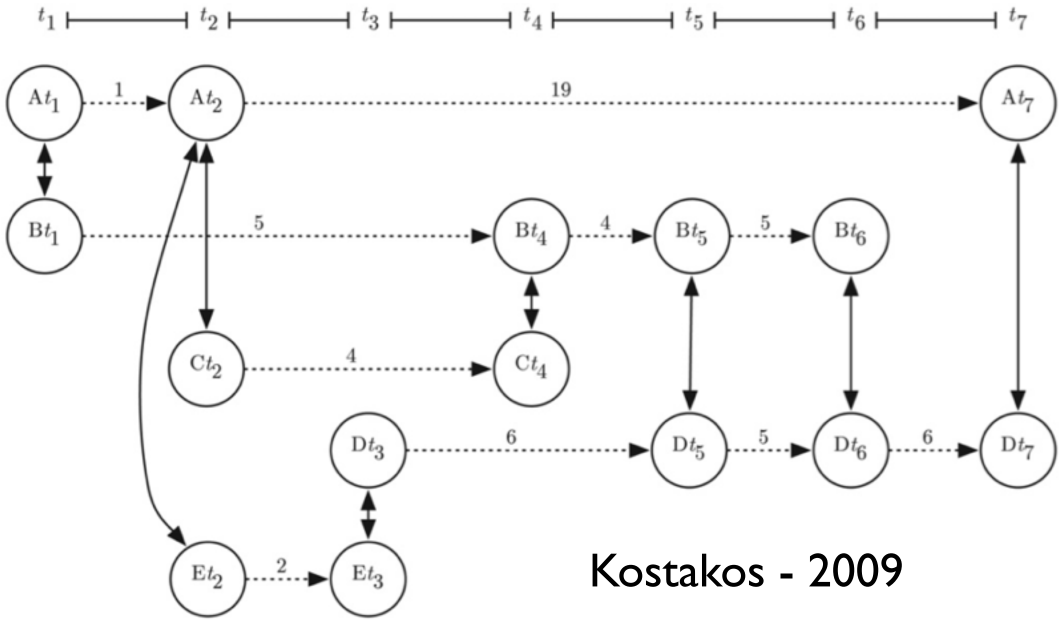
Tang et al. - 2010



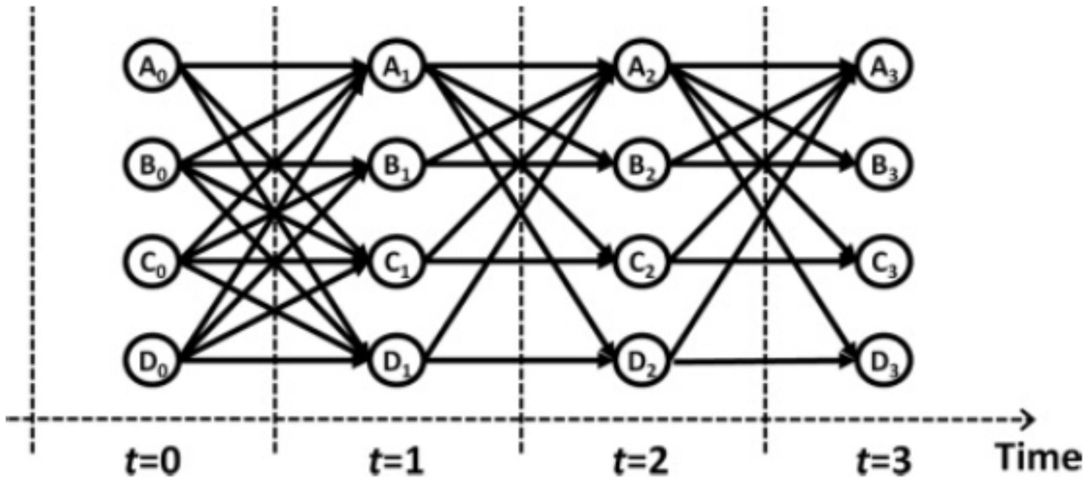
Saramäki & Holme - 2012



Casteigts et al. - 2012



Kostakos - 2009



Kim & Anderson - 2012

A Unifying Model for Representing Time-Varying Graphs

Joint work with Klaus Wehmuth (LNCC)
and Éric Fleury (ENS-Lyon/INRIA, France)

G = (V, E, T)

V - **Vertices (nodes)** set

E - **Dynamic Edges** set

T - **Time instants** set

K. Wehmuth, A. Ziviani, E. Fleury, **A Unifying Model for Representing Time-Varying Graphs**,
Technical report INRIA RR-8466, 39 p., January 2014.
Under submission. Available at http://arxiv.org/abs/1402.3488

Dynamic Edges

$E \subseteq V \times T \times V \times T$

$e \in E, \;\; e = (v_1, t_a, v_2, t_b)$

$v_1, v_2 \in V$ - vertices $t_a, t_b \in T$ - time instants

A **dynamic edge** expresses a relation between two nodes at two time instants

Dynamic Edges

$E \subseteq V \times T \times V \times T$

$e \in E,\ \ e = (v_1, t_a, v_2, t_b)$

Are represented by an **ordered quadruple**

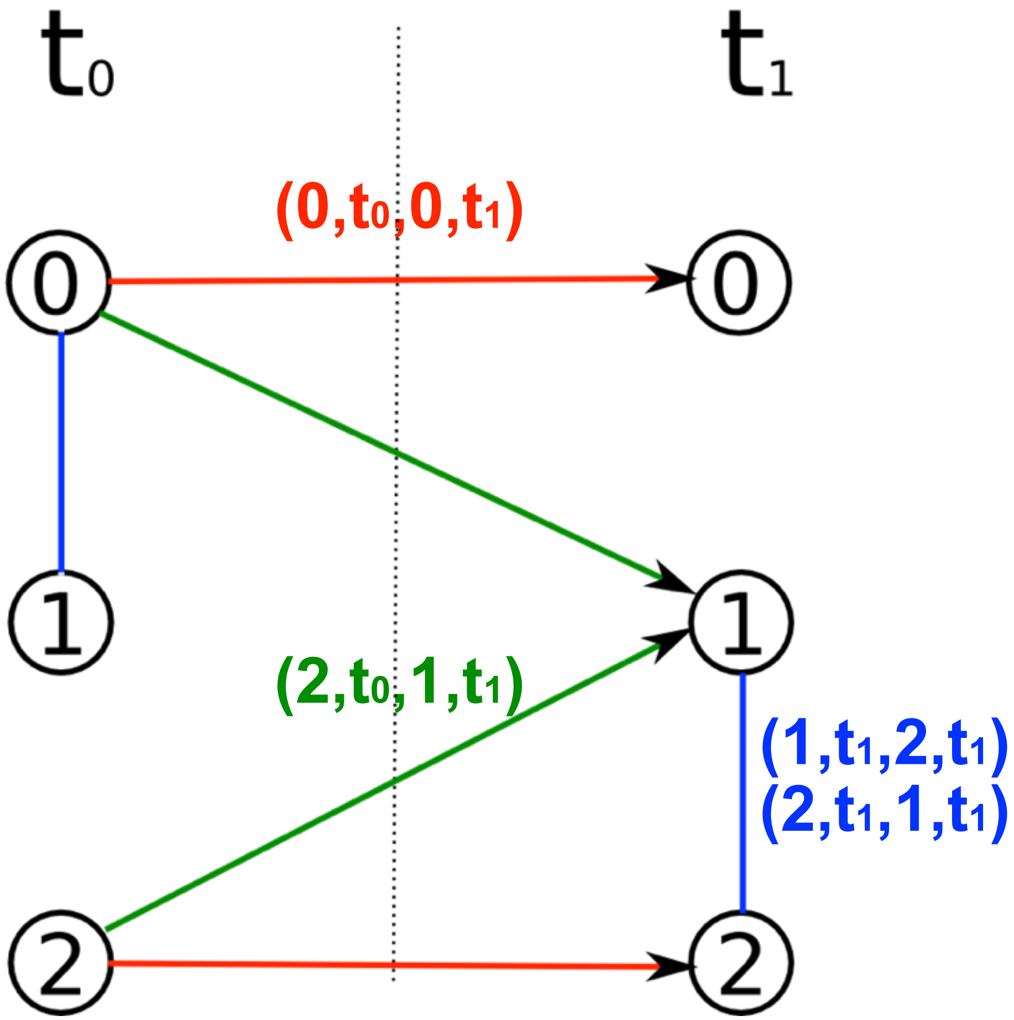
Can be represented by an entry on a
4th order tensor

Dynamic Edges

Temporal Edges

Spatial Edges

Mixed Edges



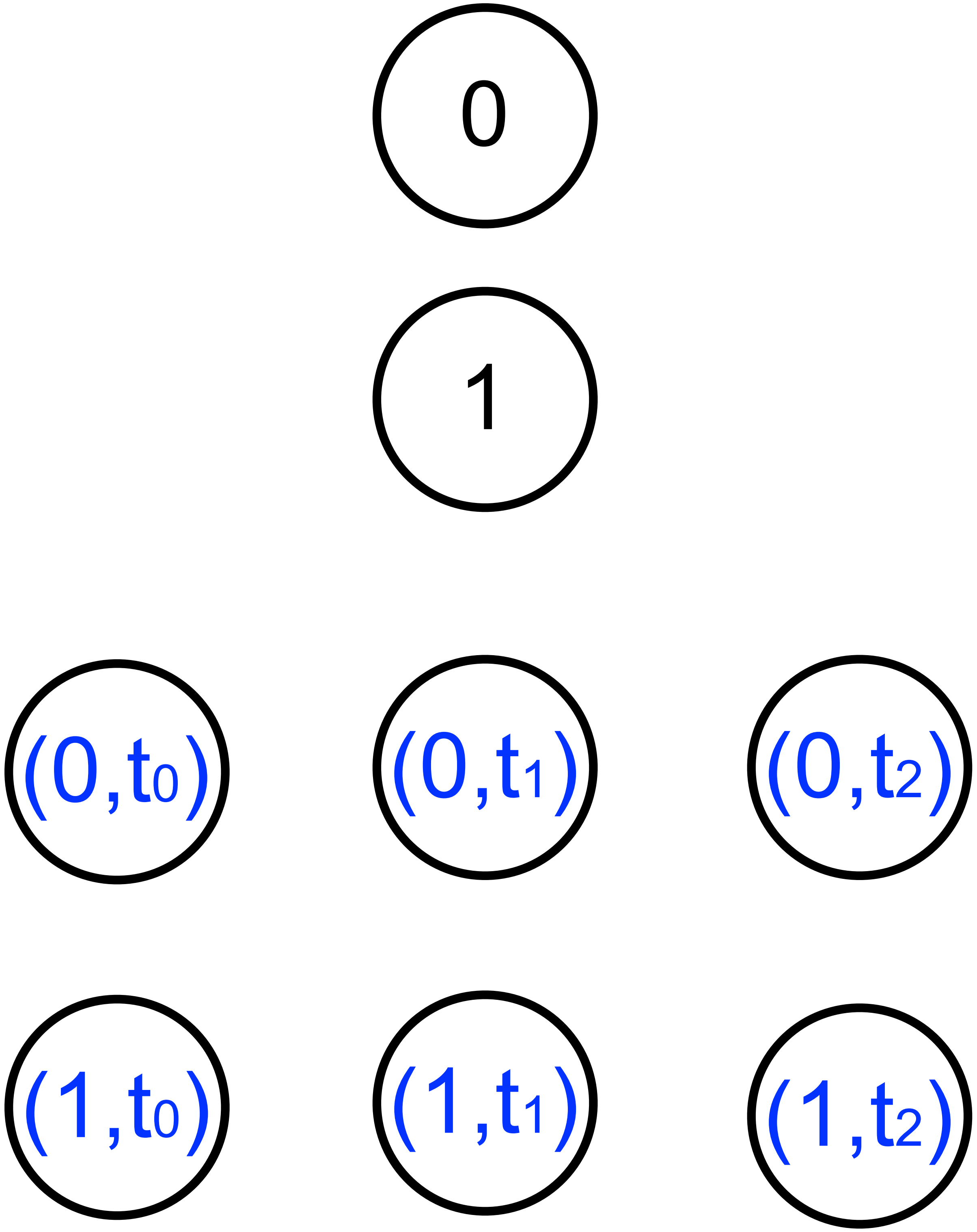
Temporal Vertices

$G = (V, E, T)$

$V = \{0, 1\}$

$T = \{t_0, t_1, t_2\}$

$\mathbf{v} \in VT = V \times T$



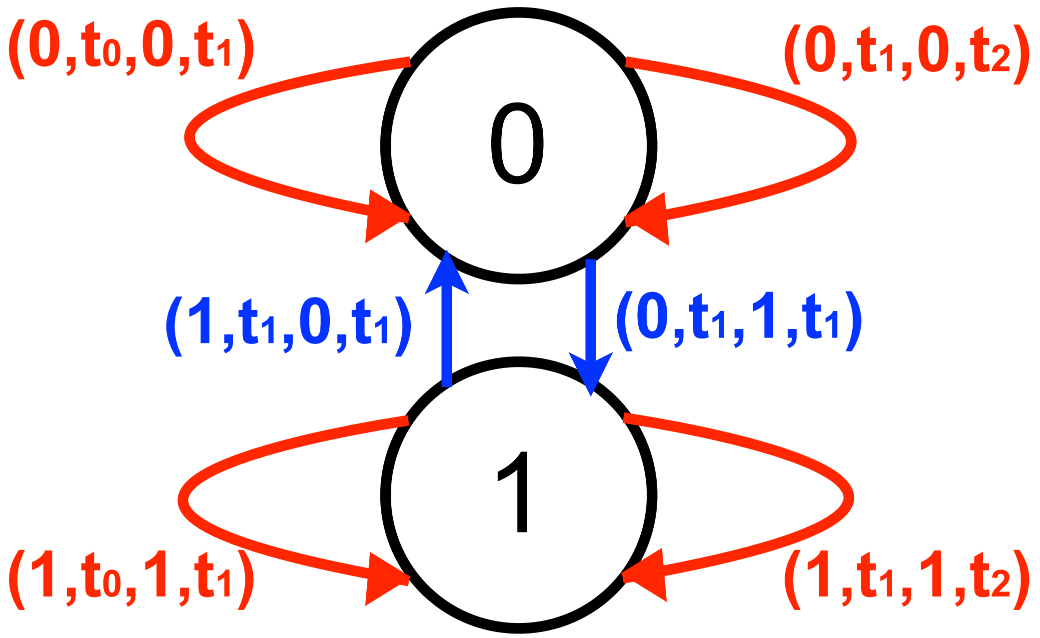
Temporal Vertices Representation

$G = (V,E,T)$

$e = (\underbrace{v_1, t_a}_{\mathbf{u}}, \underbrace{v_2, t_b}_{\mathbf{v}})$

$\mathbf{u},\mathbf{v} \in VT = V \times T$

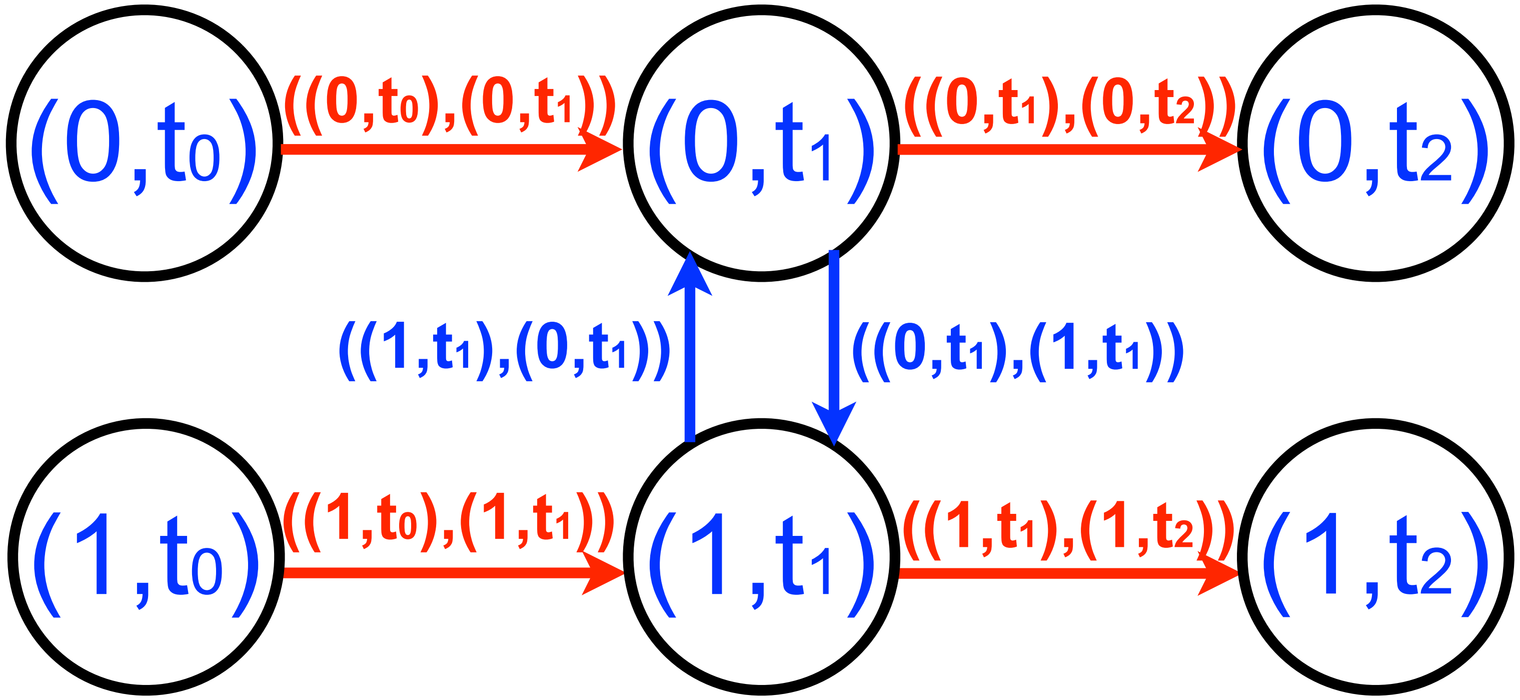
$e_s = (\mathbf{u},\mathbf{v}) \in VT \times VT = ET$



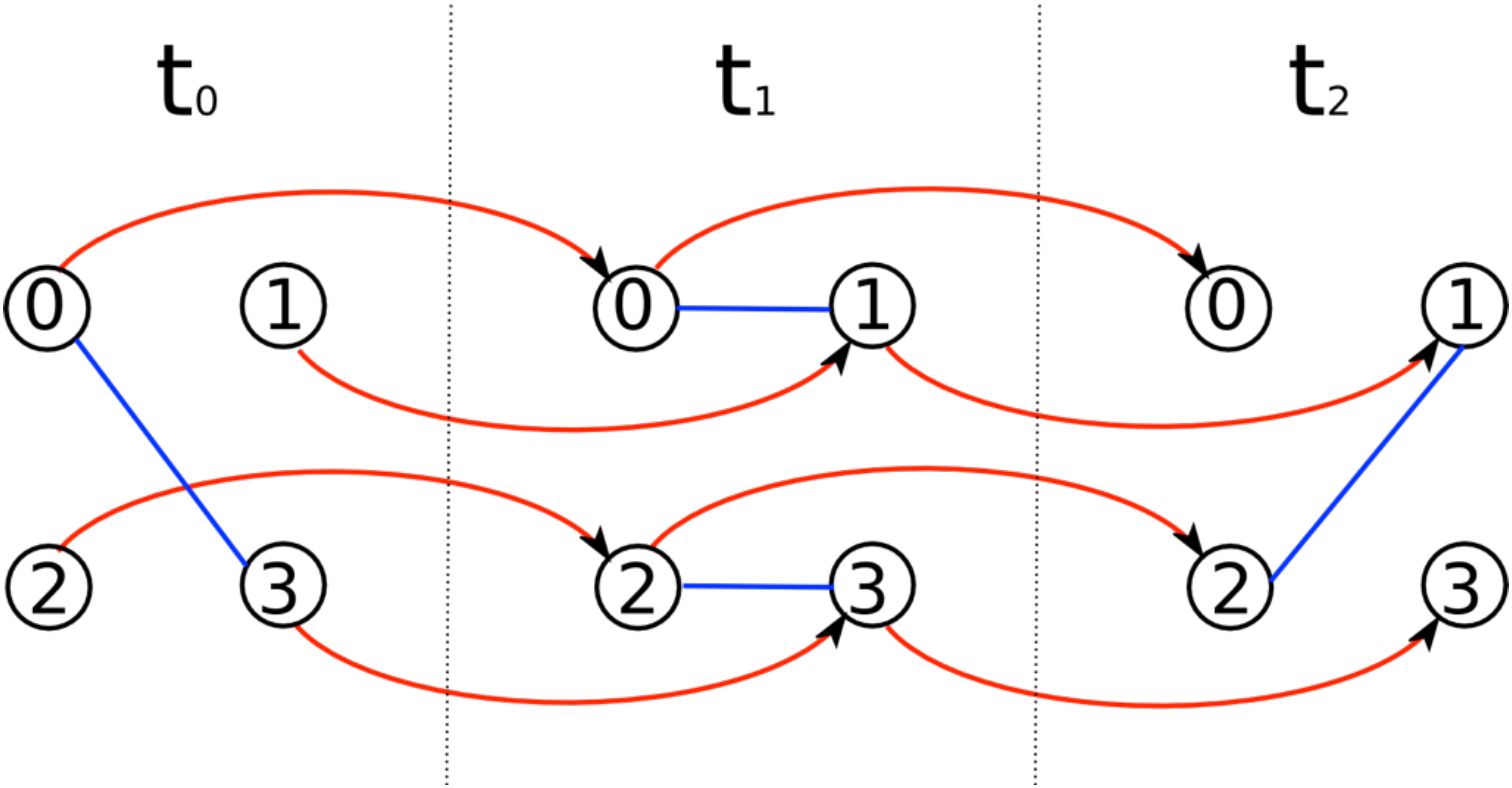
$G_s = (VT, ET)$

$I{:}V \times T \rightarrow VT$

$e_s \in ET \longleftrightarrow e \in E$



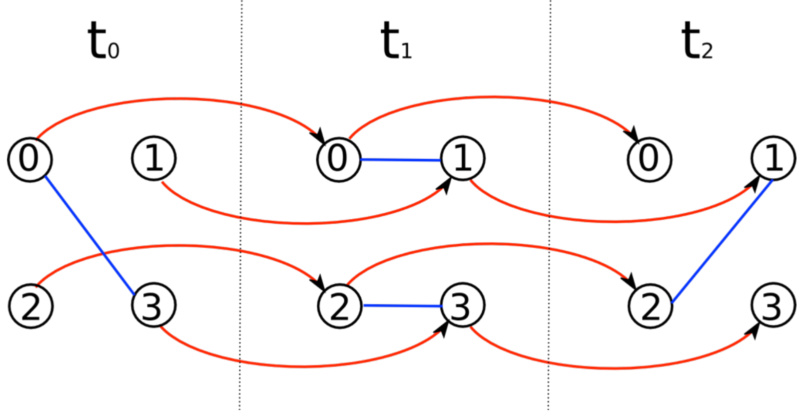
TVG Example



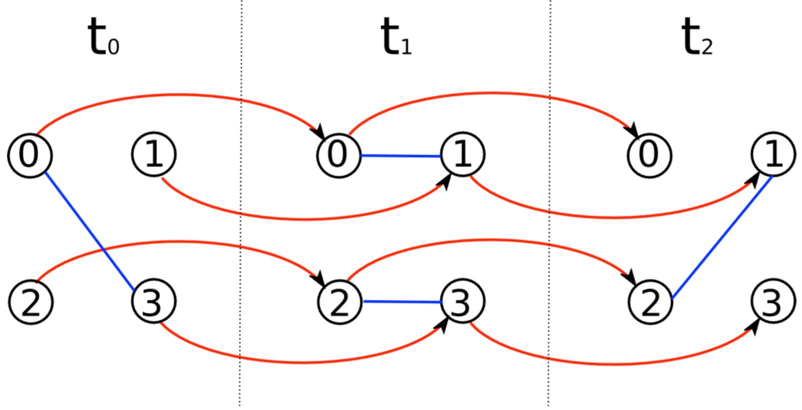
Adjacency Tensor in matrix form

$$Mat(\mathbf{A}_{G1}) = \left[\begin{array}{cccc:cccc:cccc}
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hdashline
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\begin{array}{cc}
0 & \\ 1 & t_0 \\ 2 & \\ 3 & \\
0 & \\ 1 & t_1 \\ 2 & \\ 3 & \\
0 & \\ 1 & t_2 \\ 2 & \\ 3 &
\end{array}$$

$$\begin{array}{cccc:cccc:cccc}
0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 \\
 & t_0 & & & & t_1 & & & & t_2 & &
\end{array}$$



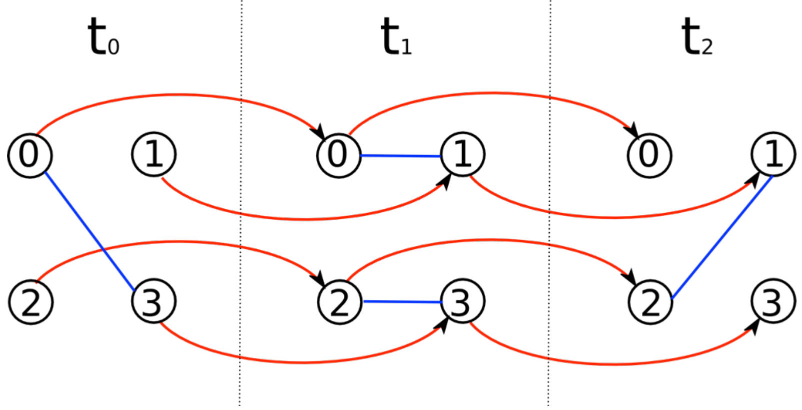
Spatial Edges



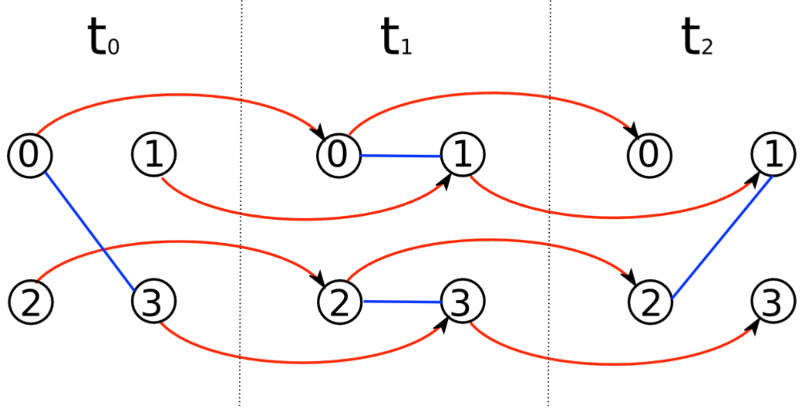
$$Mat(\mathbf{A}_{G1}) = \begin{array}{c} \left[\begin{array}{cccc|cccc|cccc} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{cc} 0 & \\ 1 & t_0 \\ 2 & \\ 3 & \\ \hline 0 & \\ 1 & t_1 \\ 2 & \\ 3 & \\ \hline 0 & \\ 1 & t_2 \\ 2 & \\ 3 & \end{array} \\ \begin{array}{cccc|cccc|cccc} 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 \\ \multicolumn{4}{c|}{t_0} & \multicolumn{4}{c|}{t_1} & \multicolumn{4}{c}{t_2} \end{array} \end{array}$$

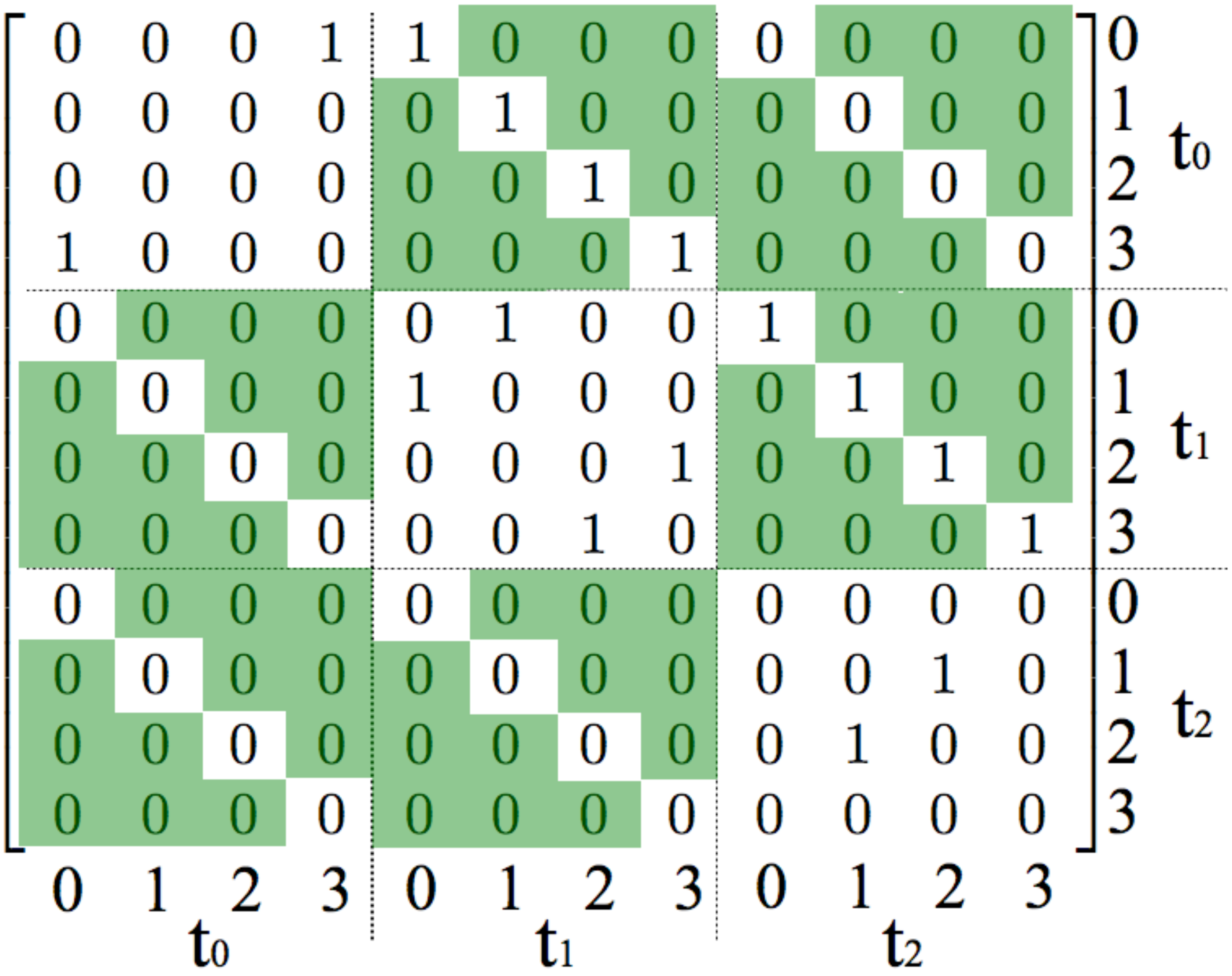
Temporal Edges

$$Mat(\mathbf{A}_{G1}) = \begin{array}{cccc|cccc|cccc|cc}
0 & 0 & 0 & 1 & \boxed{1} & 0 & 0 & 0 & \boxed{0} & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & \boxed{1} & 0 & 0 & 0 & \boxed{0} & 0 & 0 & 1 & t_0 \\
0 & 0 & 0 & 0 & 0 & 0 & \boxed{1} & 0 & 0 & 0 & \boxed{0} & 0 & 2 & \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & \boxed{1} & 0 & 0 & 0 & \boxed{0} & 3 & \\
\hline
\boxed{0} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \boxed{1} & 0 & 0 & 0 & 0 & \\
0 & \boxed{0} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \boxed{1} & 0 & 0 & 1 & t_1 \\
0 & 0 & \boxed{0} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \boxed{1} & 0 & 2 & \\
0 & 0 & 0 & \boxed{0} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \boxed{1} & 3 & \\
\hline
\boxed{0} & 0 & 0 & 0 & \boxed{0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & \boxed{0} & 0 & 0 & 0 & \boxed{0} & 0 & 0 & 0 & 0 & 1 & 0 & 1 & t_2 \\
0 & 0 & \boxed{0} & 0 & 0 & 0 & \boxed{0} & 0 & 0 & 1 & 0 & 0 & 2 & \\
0 & 0 & 0 & \boxed{0} & 0 & 0 & 0 & \boxed{0} & 0 & 0 & 0 & 0 & 3 & \\
0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & & \\
\multicolumn{4}{c|}{t_0} & \multicolumn{4}{c|}{t_1} & \multicolumn{4}{c|}{t_2} & &
\end{array}$$



Mixed Edges

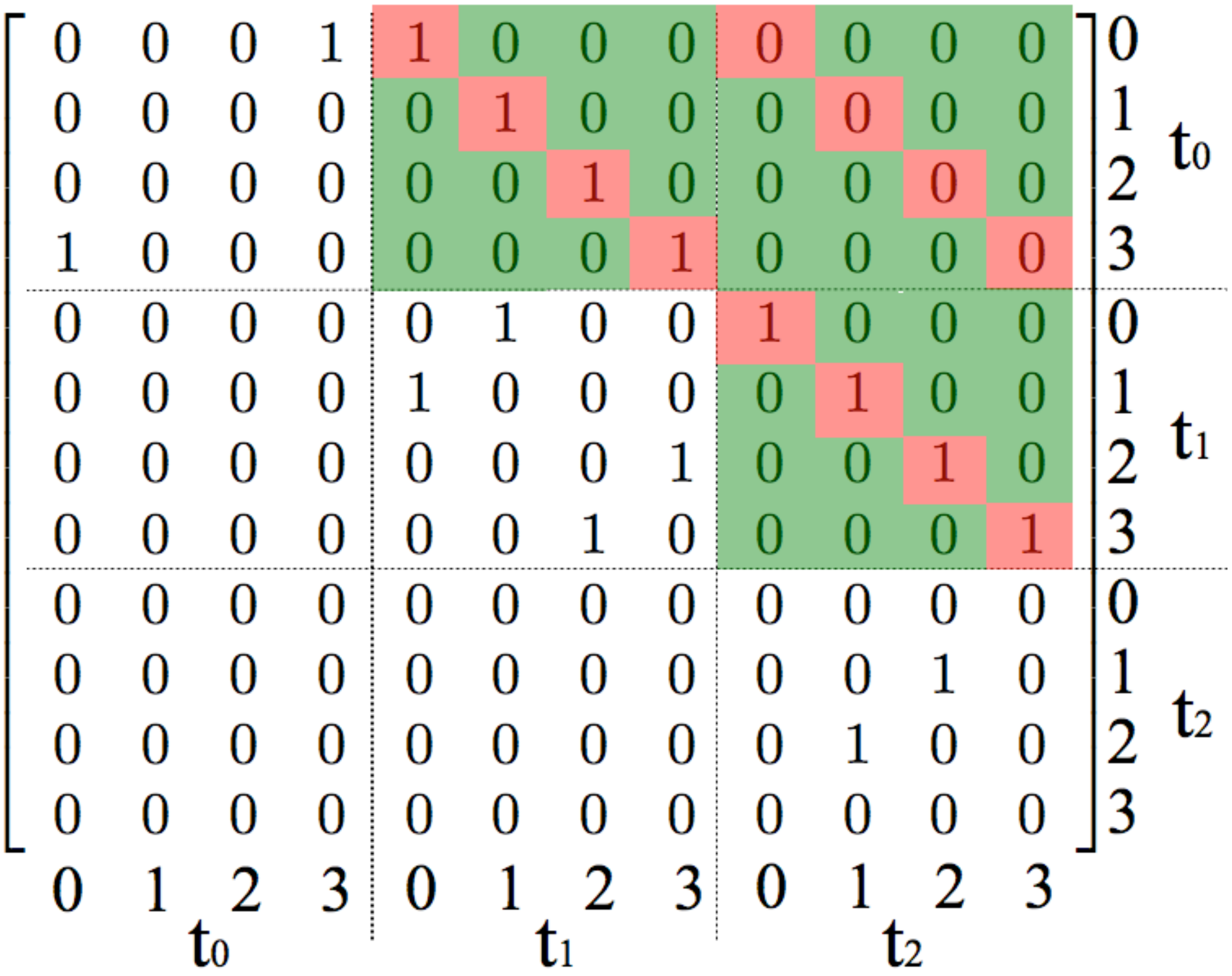




$$
Mat(\mathbf{A}_{G1}) = \left[\begin{array}{cccc:cccc:cccc}
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hdashline
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\begin{array}{cl}
0 & \\ 1 & t_0 \\ 2 & \\ 3 & \\ \hdashline
0 & \\ 1 & t_1 \\ 2 & \\ 3 & \\ \hdashline
0 & \\ 1 & t_2 \\ 2 & \\ 3 &
\end{array}
$$

$$
\begin{array}{cccc:cccc:cccc}
0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 \\
 & t_0 & & & & t_1 & & & & t_2 & &
\end{array}
$$

Progressive Edges



$$
Mat(\mathbf{A}_{G1}) = \begin{array}{cccc|cccc|cccc|cc}
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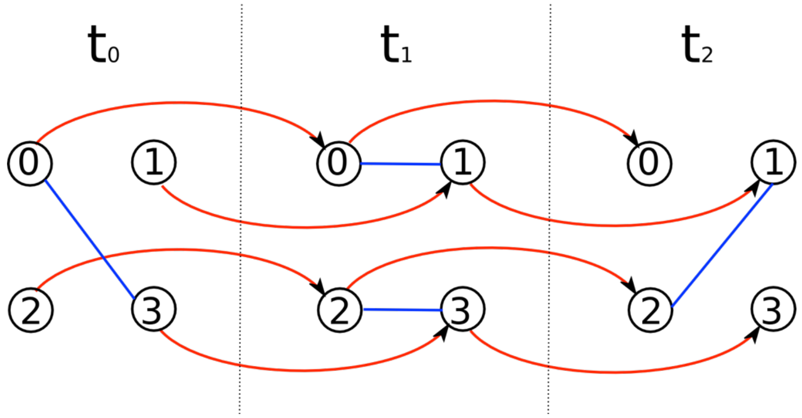
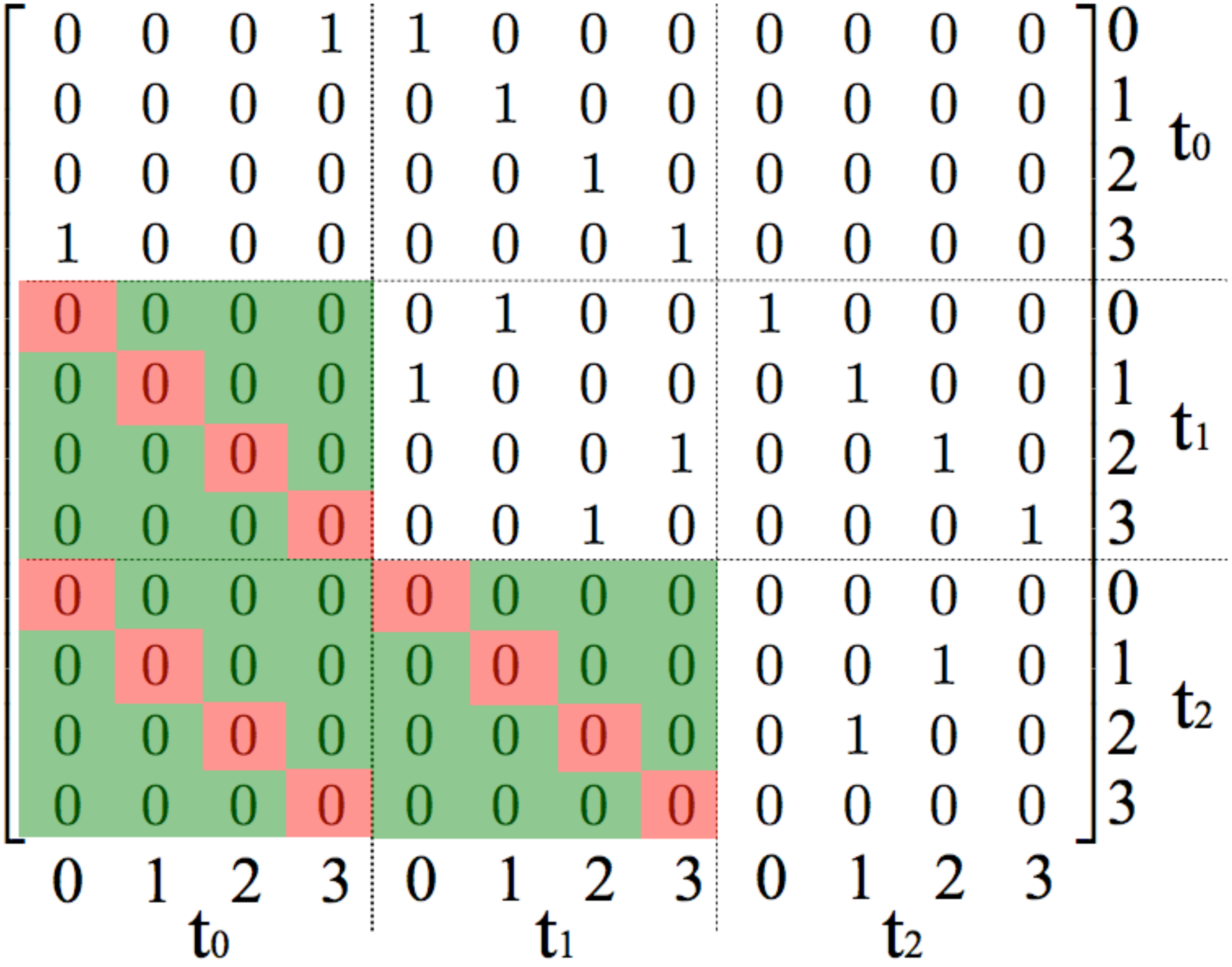
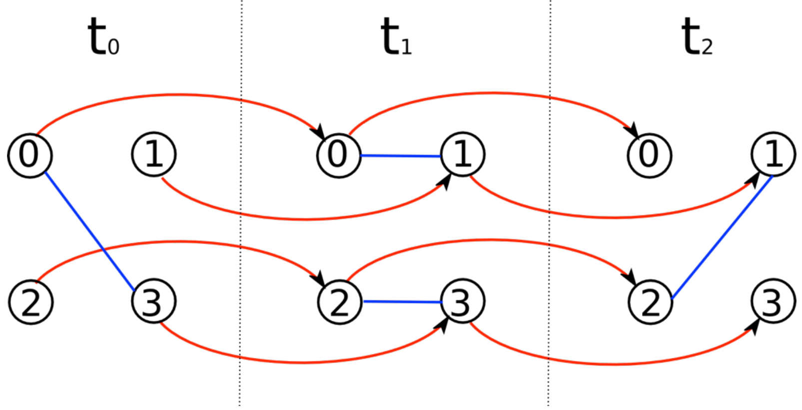
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & t_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 3 & \\
\hline
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & t_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 2 & \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 3 & \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & t_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & \\
\hline
0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & & \\
 & t_0 & & & & t_1 & & & & t_2 & & & &
\end{array}
$$



Regressive Edges



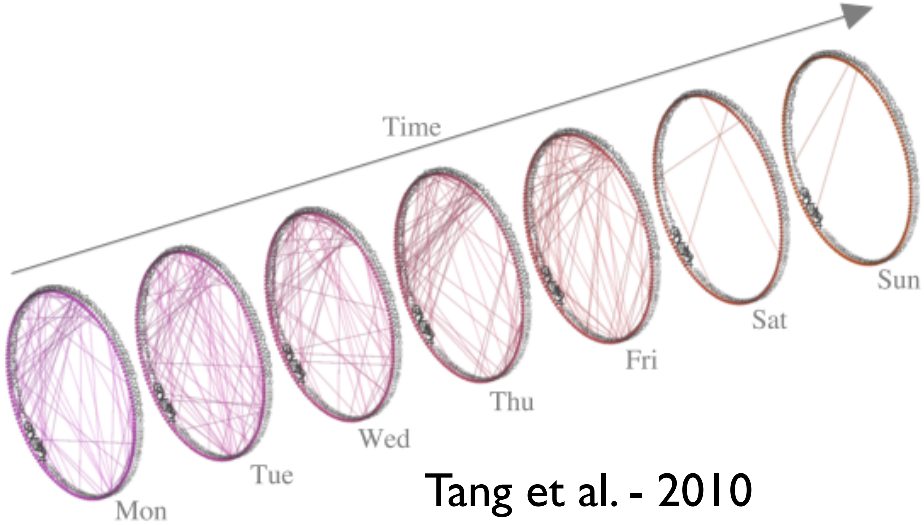
$$Mat(\mathbf{A}_{G1}) = \begin{array}{cccc:cccc:cccc|cc}
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & t_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 2 & \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 3 & \\
\hdashline
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & t_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 2 & \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 3 & \\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & t_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & \\
\hline
0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & & \\
 & t_0 & & & & t_1 & & & & t_2 & & & &
\end{array}$$



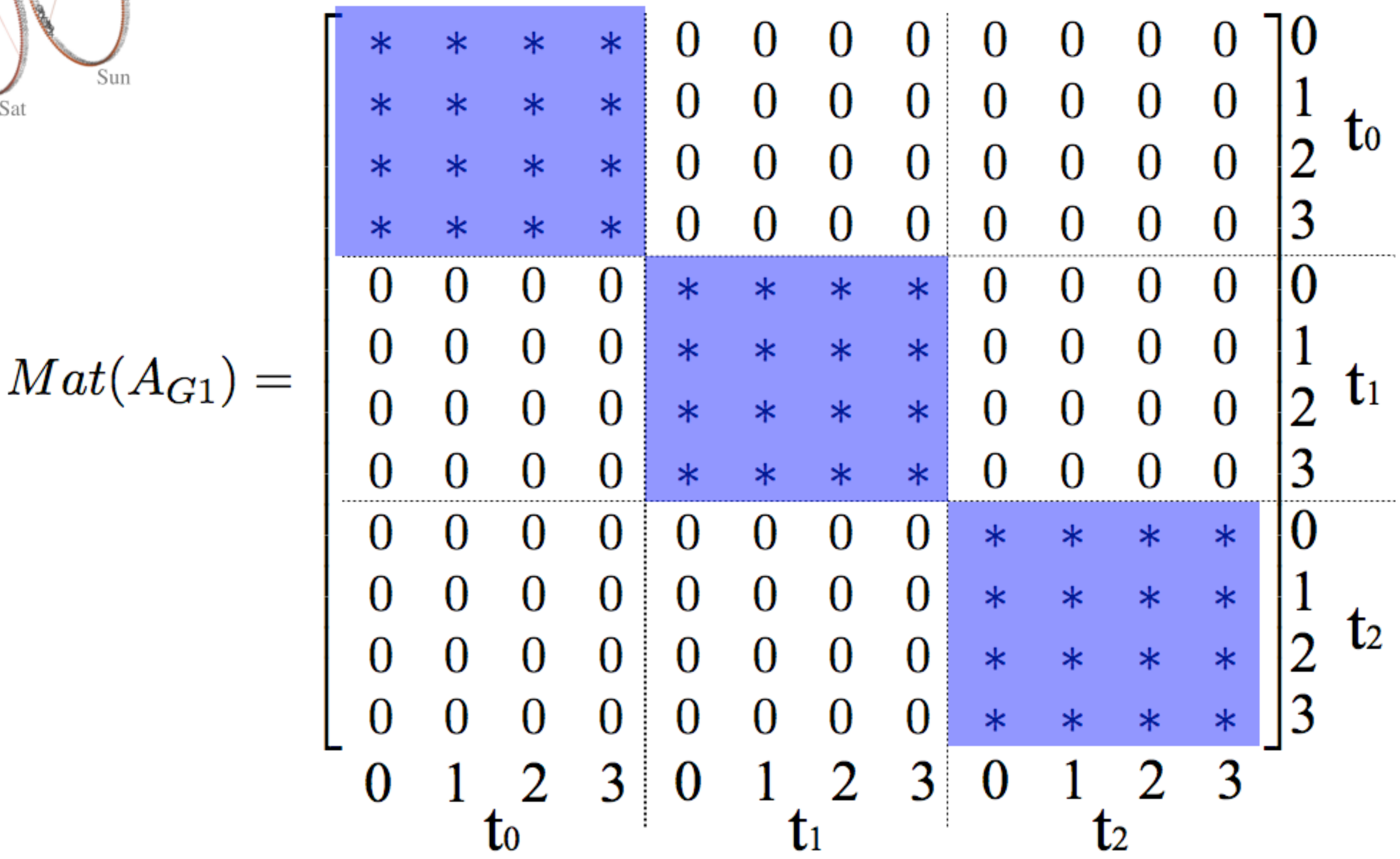
Our proposed model unifies previous TVG models

i.e. it can represent previous TVG models, which can not necessarily represent each other

Snapshot Models

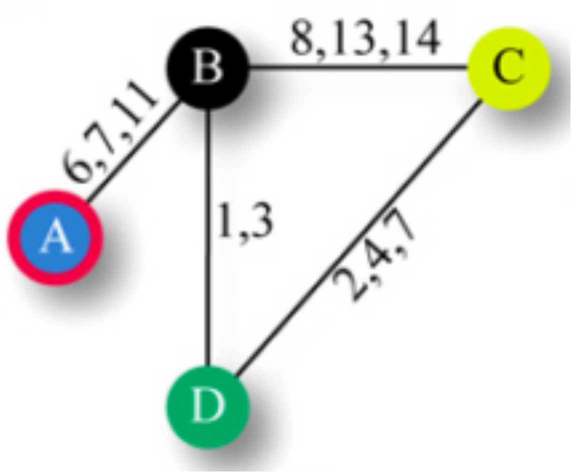


Tang et al. - 2010



$$Mat(A_{G1}) = \left[\begin{array}{cccc|cccc|cccc}
* & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & * & * & * & * & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & * & * & * & * & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & * & * & * & * & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & * & * & * & * & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & *
\end{array}\right]
\begin{array}{cc}
0 & \\ 1 & t_0 \\ 2 & \\ 3 & \\
0 & \\ 1 & t_1 \\ 2 & \\ 3 & \\
0 & \\ 1 & t_2 \\ 2 & \\ 3 &
\end{array}$$

$$\begin{array}{cccc|cccc|cccc}
0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 \\
 & t_0 & & & & t_1 & & & & t_2 & &
\end{array}$$

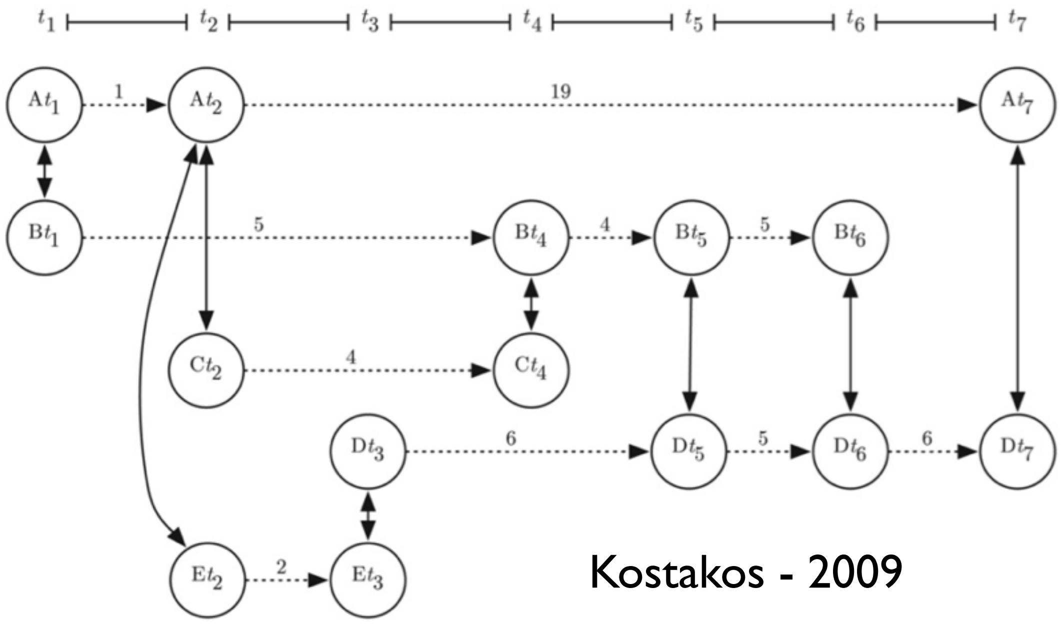


Saramäki & Holme - 2012

Kostakos 2009

nodes can connect to themselves in distinct time instants

$$Mat(A_{G1}) = \begin{array}{c|c}
\left[\begin{array}{cccc:cccc:cccc}
\ast & \ast & \ast & \ast & \ast & 0 & 0 & 0 & \ast & 0 & 0 & 0 \\
\ast & \ast & \ast & \ast & 0 & \ast & 0 & 0 & 0 & \ast & 0 & 0 \\
\ast & \ast & \ast & \ast & 0 & 0 & \ast & 0 & 0 & 0 & \ast & 0 \\
\ast & \ast & \ast & \ast & 0 & 0 & 0 & \ast & 0 & 0 & 0 & \ast \\
\hdashline
0 & 0 & 0 & 0 & \ast & \ast & \ast & \ast & \ast & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \ast & \ast & \ast & \ast & 0 & \ast & 0 & 0 \\
0 & 0 & 0 & 0 & \ast & \ast & \ast & \ast & 0 & 0 & \ast & 0 \\
0 & 0 & 0 & 0 & \ast & \ast & \ast & \ast & 0 & 0 & 0 & \ast \\
\hdashline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ast & \ast & \ast & \ast \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ast & \ast & \ast & \ast \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ast & \ast & \ast & \ast \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ast & \ast & \ast & \ast
\end{array}\right]
&
\begin{array}{cc}
0 & \\ 1 & \\ 2 & t_0 \\ 3 & \\ \hdashline
0 & \\ 1 & \\ 2 & t_1 \\ 3 & \\ \hdashline
0 & \\ 1 & \\ 2 & t_2 \\ 3 &
\end{array}
\\
\begin{array}{cccc:cccc:cccc}
0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 \\
 & t_0 & & & & t_1 & & & & t_2 & &
\end{array}
&
\end{array}$$

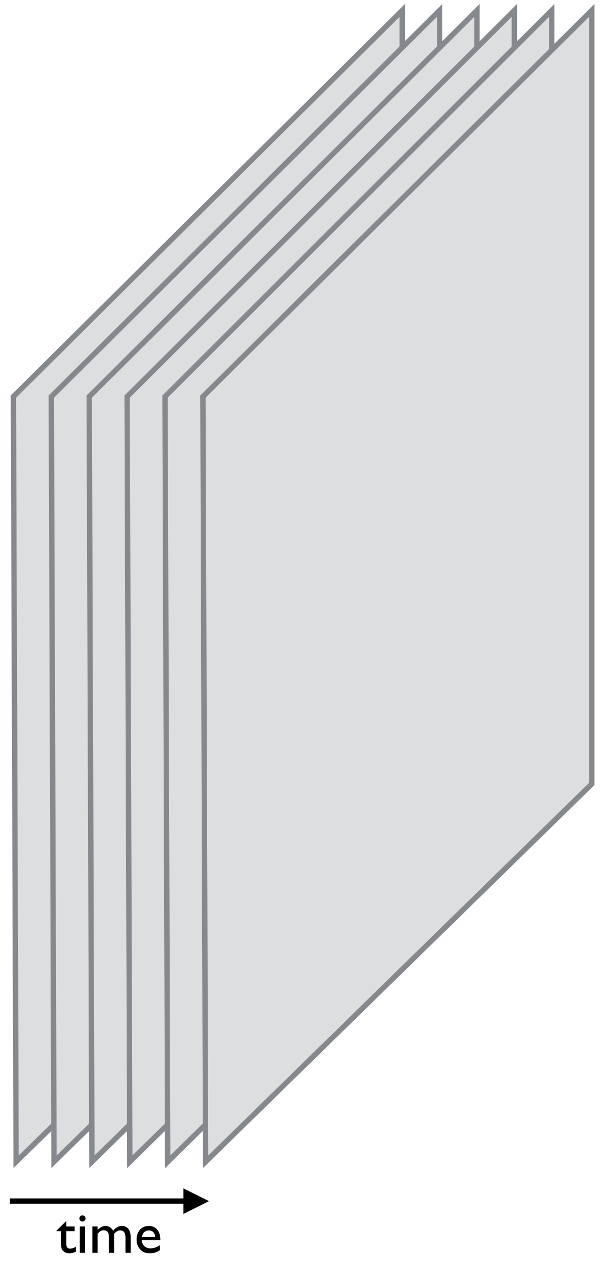


Kostakos - 2009

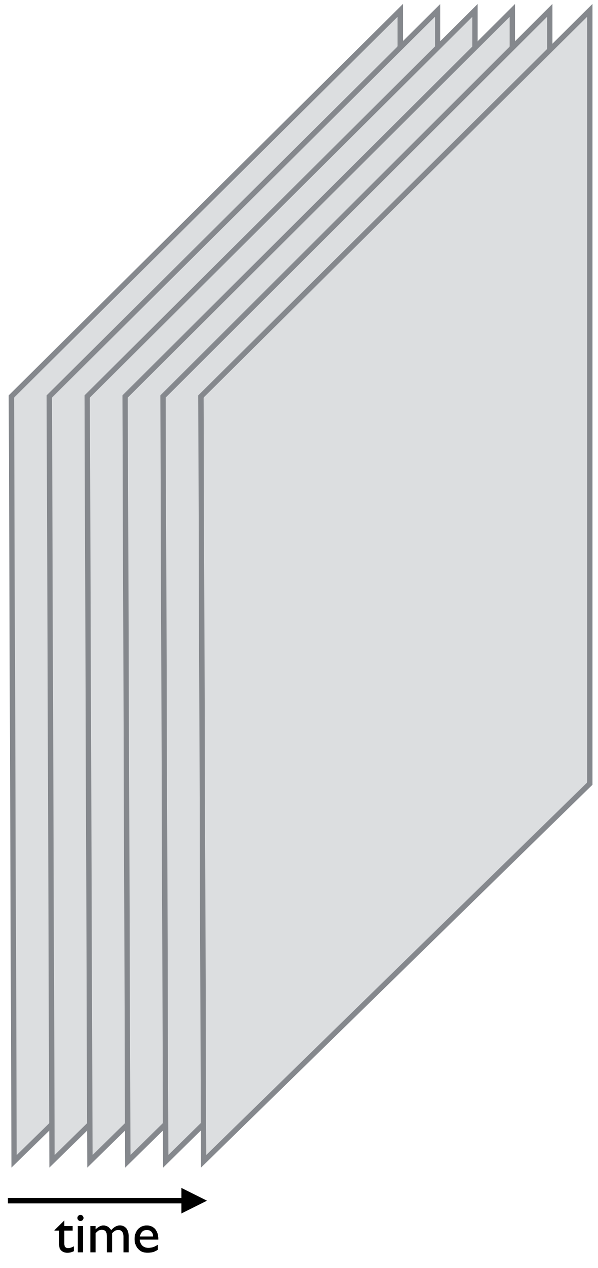
From TVGs to MultiAspect Graphs (MAGs)

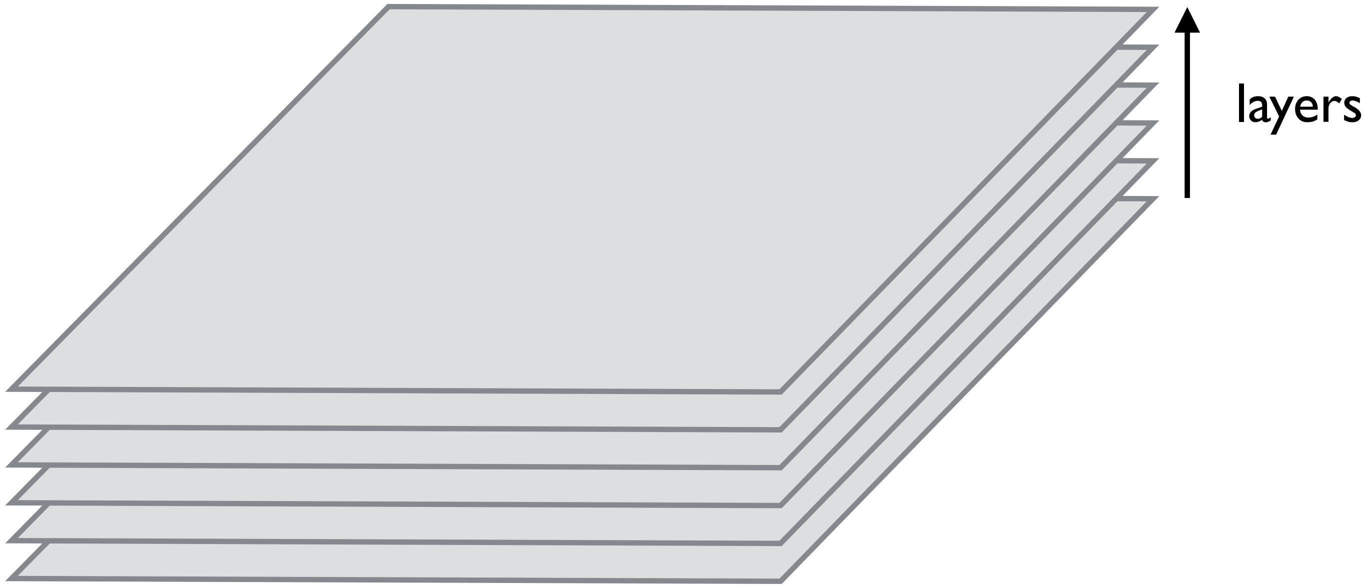
Joint work with Klaus Wehmuth (LNCC)
and Éric Fleury (ENS-Lyon/INRIA, France)

K. Wehmuth, E. Fleury, A. Ziviani, On MultiAspect Graphs,
August 2014. Under submission. Available in **http://arxiv.org/abs/1408.0943**



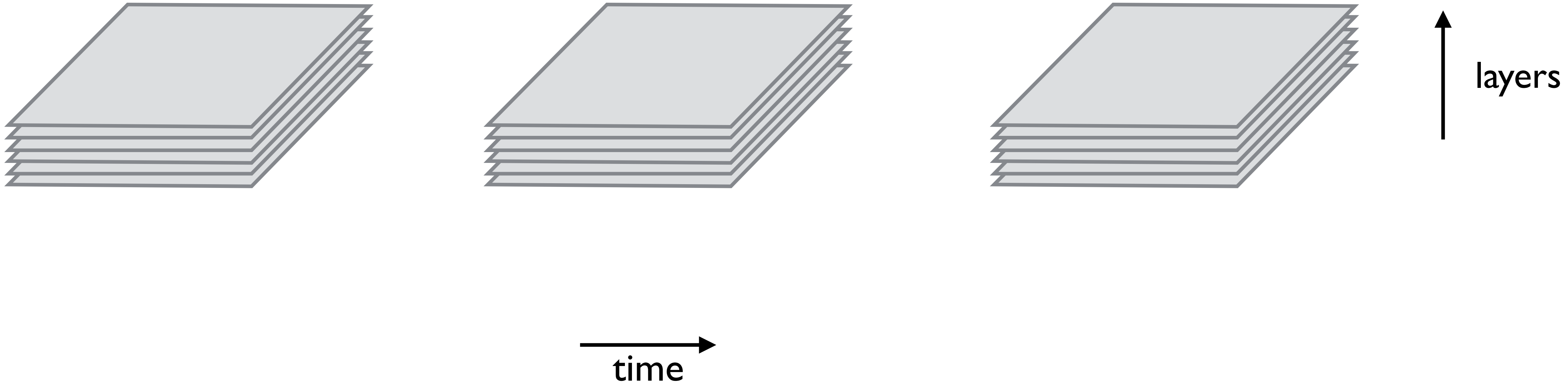
TVGs: vertices and time instants as key features





multilayer networks:
vertices and layers
as key features

TVGs: vertices and time instants as key features



time-varying multilayer networks:
vertices, time instants, layers
as key features

MultiAspect Graph (MAG)

G = (A,E)

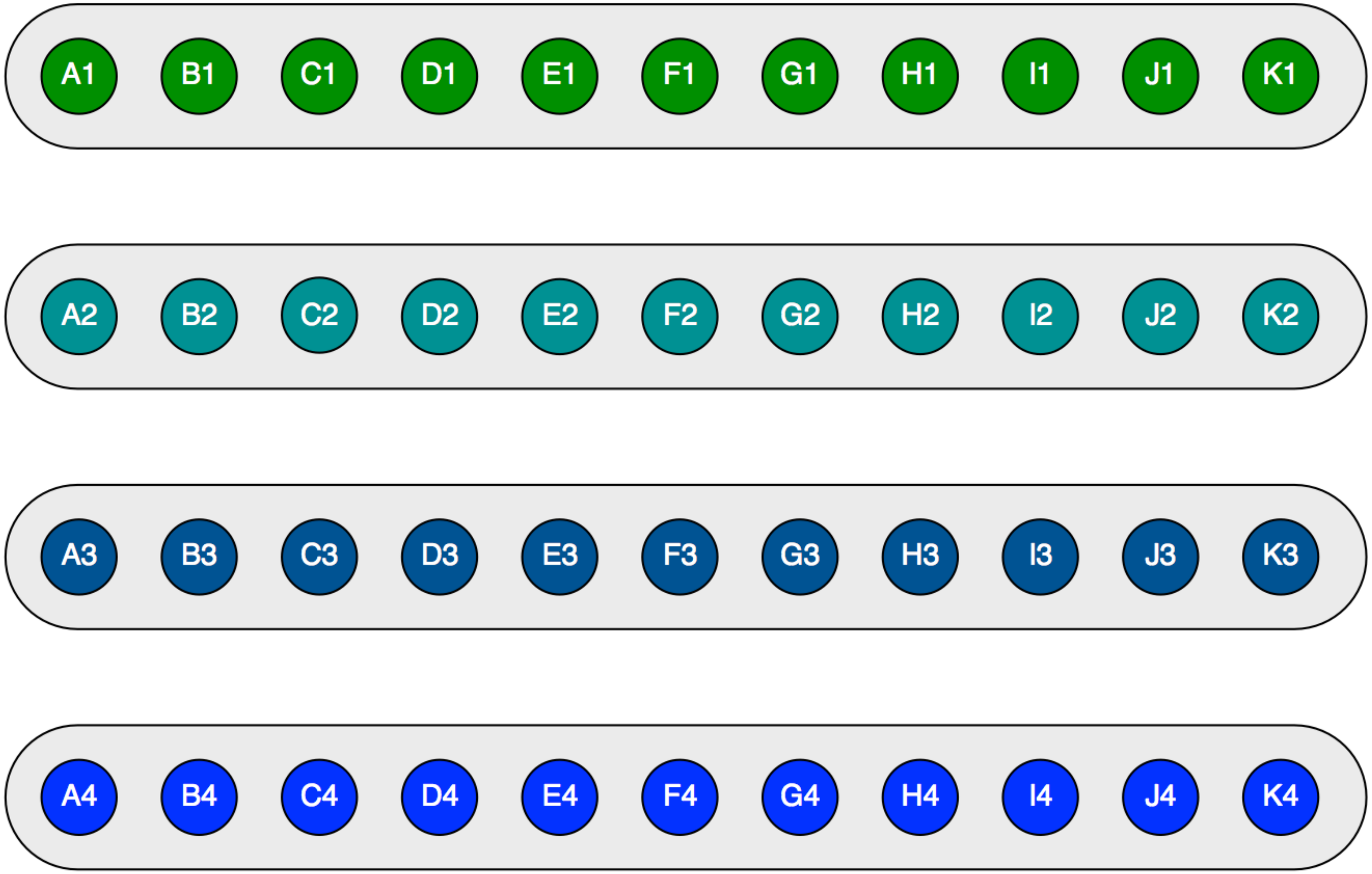
A : list of **aspects**

E : set of edges

An **aspect** is a finite set of **independent features** (vertices, time instants, layers, …)

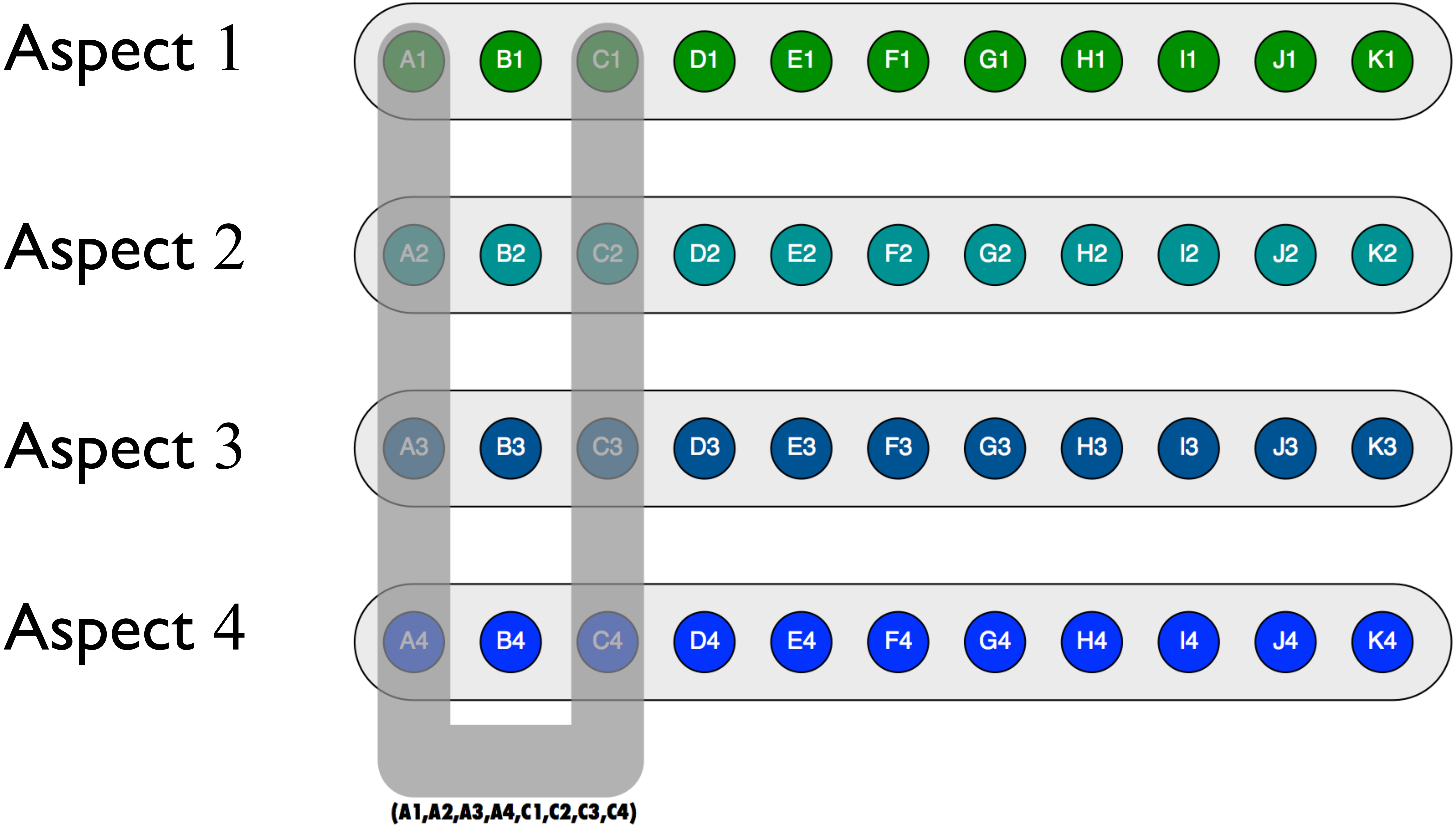
A MAG edge is a tuple of aspect elements (indicating a relation between groups of aspects)

MAG Aspects

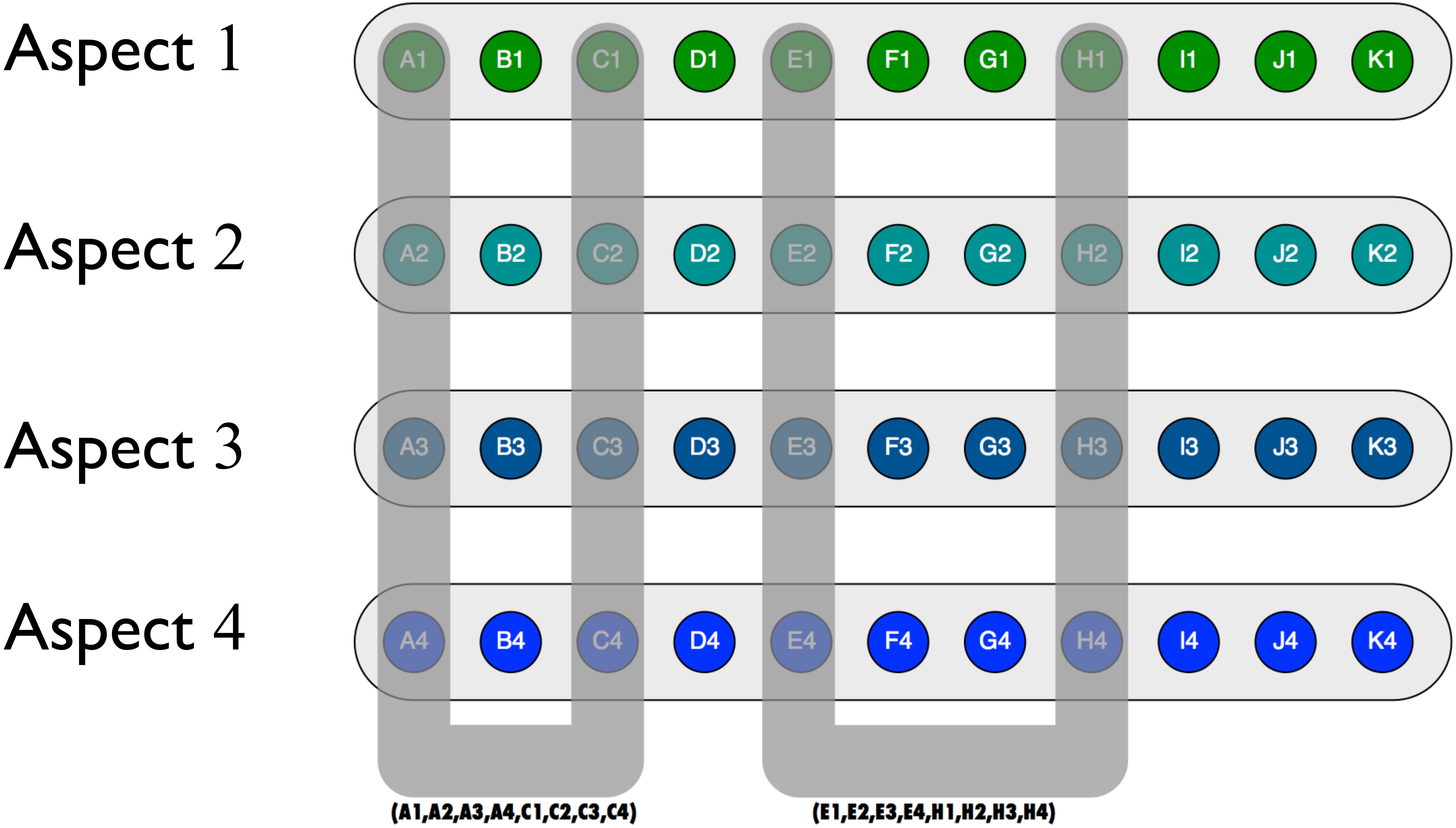


Aspect 1	A1	B1	C1	D1	E1	F1	G1	H1	I1	J1	K1
Aspect 2	A2	B2	C2	D2	E2	F2	G2	H2	I2	J2	K2
Aspect 3	A3	B3	C3	D3	E3	F3	G3	H3	I3	J3	K3
Aspect 4	A4	B4	C4	D4	E4	F4	G4	H4	I4	J4	K4

MAG Edges


Aspect 1
Aspect 2
Aspect 3
Aspect 4
A1
B1
C1
D1
E1
F1
G1
H1
I1
J1
K1
A2
B2
C2
D2
E2
F2
G2
H2
I2
J2
K2
A3
B3
C3
D3
E3
F3
G3
H3
I3
J3
K3
A4
B4
C4
D4
E4
F4
G4
H4
I4
J4
K4
(A1,A2,A3,A4,C1,C2,C3,C4)

MAG Edges



Aspect 1

Aspect 2

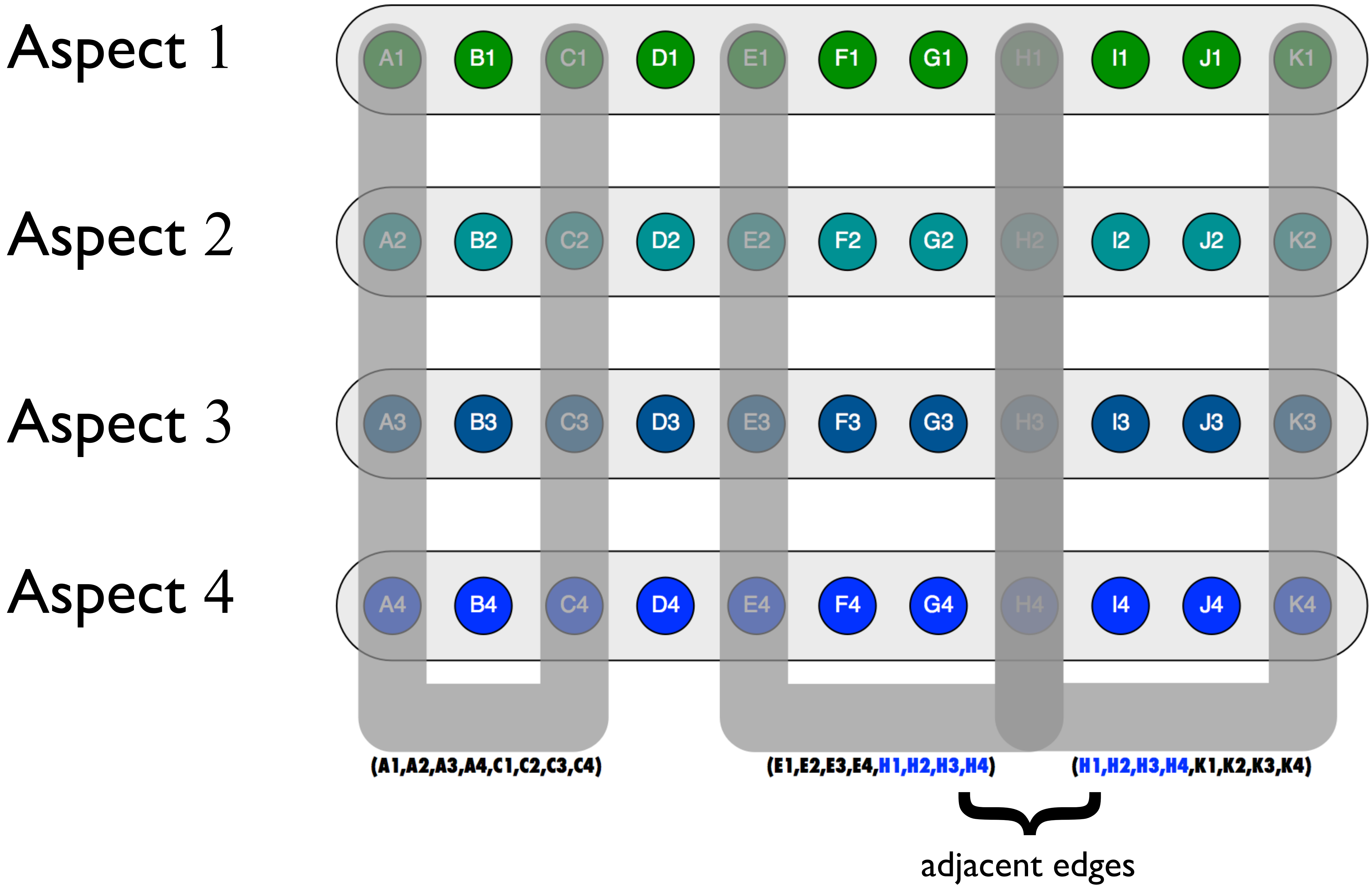
Aspect 3

Aspect 4

(A1,A2,A3,A4,C1,C2,C3,C4)

(E1,E2,E3,E4,H1,H2,H3,H4)

MAG Edges



Aspect 1

Aspect 2

Aspect 3

Aspect 4

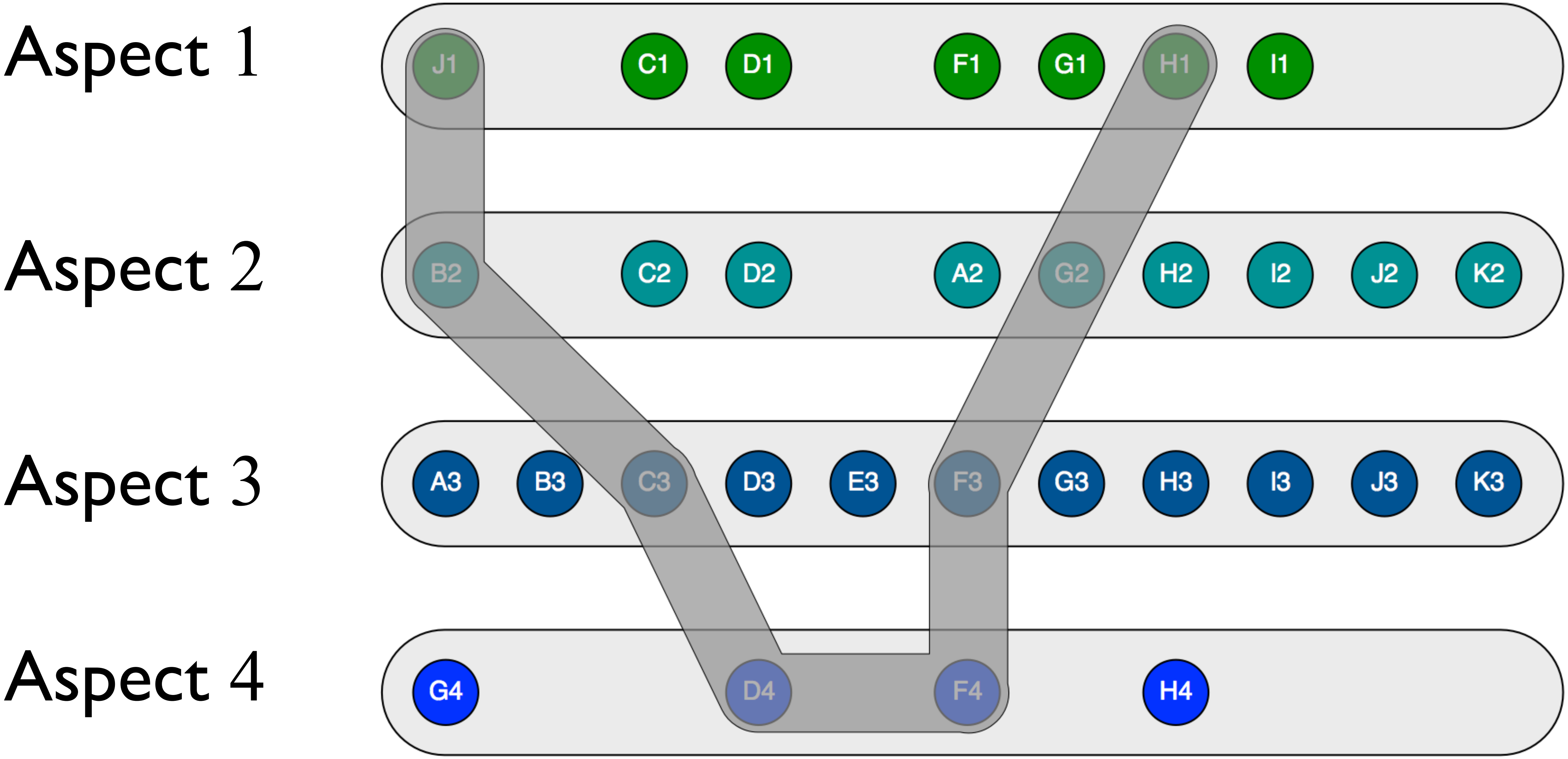
(A1,A2,A3,A4,C1,C2,C3,C4)

(E1,E2,E3,E4,H1,H2,H3,H4)

(H1,H2,H3,H4,K1,K2,K3,K4)

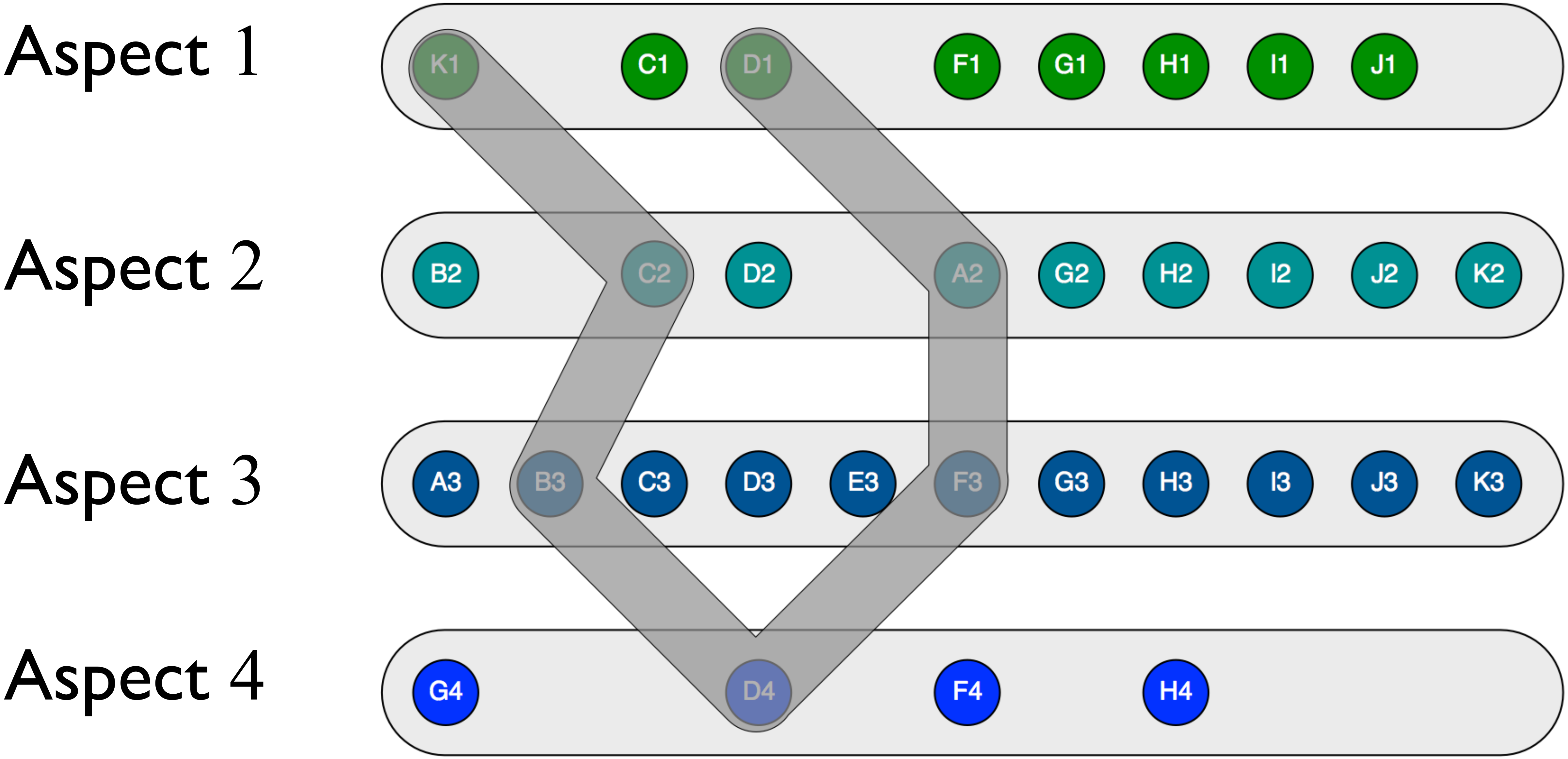
adjacent edges

MAG Edges



(J1,B2,C3,D4,H1,G2,F3,F4)

MAG Edges



(K1,C2,B3,D4,D1,A2,F3,D4)

MAG illustrative example

Multimodal transportation system as a 3-aspect MAG

- A[1] - first aspect - vertices
  - bus stops, subway stations, …
- A[2] - second aspect - layers of transportation networks
  - bus network, subway network, …
- A[3] - third aspect - schedule - departure and arrival

An edge $(bs_1, bus, t_a, bs_2, bus, t_b)$ could represent a bus leaving bus stop bs_1 at time t_a and arriving to bus stop bs_2 at time t_b.

MAG Edges

$e = (bs_1, bus, t_a, bs_2, bus, t_b)$ is an edge from previous example

The number of elements in an edge is always even and equal to twice the number of aspects

$$(\underbrace{bs_1, bus, t_a}_{\text{origin}}\ ,\ \underbrace{bs_2, bus, t_b}_{\text{destination}})$$

Composite Vertices

From the edge structure, it follows that the isomorphism to traditional oriented graphs found for TVGs can be extended to MAGs

Final remarks

The structure of a MAG is similar to an **even uniform hypergraph**

Nevertheless, the **additional structure** imposed upon the way an edge is constructed on a MAG employs the concept of even uniform hypergraphs in a fashion which **better suits** the intended application of modelling multi-layered dynamic networked systems

Similar idea with some variations appeared recently in Kivelä et al. “Multilayer Networks”, Journal of Complex Networks (Oxford Press), latest version 18-Aug-2014

Final remarks: MAG Applications

MultiAspect Graph G = (A,E)

IAI	Composite Vertex	Edge	Examples
1	simple objects	pairs	traditional directed graph
2	pairs	quadruples	TVG; multilayer graph
3	triples	sextuples	time-varying multilayer graph
4	quadruples	octuples	…

K. Wehmuth, E. Fleury, A. Ziviani, On MultiAspect Graphs,
August 2014. Under submission.. Available in http://arxiv.org/abs/1408.0943

On-going work

- Algebraic MAG representation and algorithms
- Centrality on MAGs

Thanks!

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CAPES

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Finep INOVAÇÃO E PESQUISA

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